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IN CO-OPERATION WITH THE OTHER MEMBERS OF THE ECONOMICS DEPARTMENT OF THE UNIVERSITY OF CHICAGO

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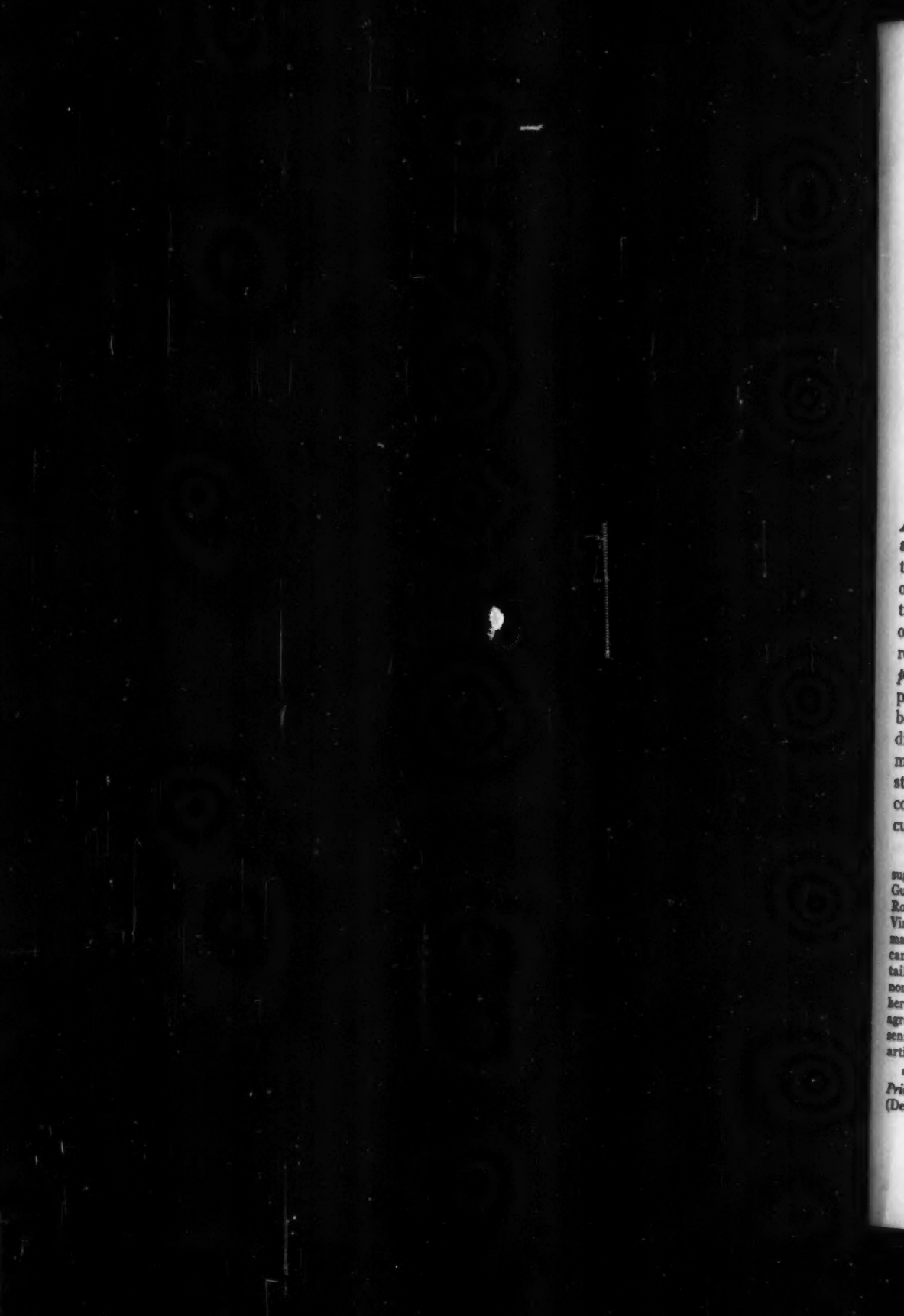

THE JOURNAL OF
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THE MARSHALLIAN DEMAND CURVE

MILTON FRIEDMAN[1]

ALFRED MARSHALL'S theory of demand strikingly exemplifies his "impatience with rigid definition and an excessive tendency to let the context explain his meaning."[2] The concept of the demand curve as a functional relation between the quantity and the price of a particular commodity is explained repeatedly and explicitly in the *Principles of Economics:* in words in the text, in plane curves in the footnotes, and in symbolic form in the Mathematical Appendix. A complete definition of the demand curve, including, in particular, a statement of the variables that are to be considered the same for all points on the curve and the variables that are to be allowed to vary, is nowhere given explicitly. The reader is left to infer the contents of *ceteris paribus* from general and vague statements, parenthetical remarks, examples that do not purport to be exhaustive, and concise mathematical notes in the Appendix.

In view of the importance of the demand curve in Marshallian analysis, it is natural that other economists should have constructed a rigorous definition to fill the gap that Marshall left. This occurred at an early date, apparently without controversy about the interpretation to be placed on Marshall's comments. The resulting definition of the demand curve is now so much an intrinsic part of current economic theory and is so widely accepted as Marshall's own that the assertion that Marshall himself gave no explicit rigorous definition may shock most readers.

Yet why this particular interpretation evolved and why it gained such unquestioned acceptance are a mystery that requires explanation. The currently accepted interpretation can be read into Marshall only by a liberal—and, I think, strained—reading of his remarks, and its acceptance implicitly convicts him of log-

[1] I am deeply indebted for helpful criticism and suggestions to A. F. Burns, Aaron Director, C. W. Guillebaud, H. Gregg Lewis, A. R. Prest, D. H. Robertson, G. J. Stigler, and, especially, Jacob Viner, to whose penetrating discussion of the demand curve in his course in economic theory I can trace some of the central ideas and even details of this article. The standard comment that none is to be held responsible for the views expressed herein has particular relevance, since most disagreed with my interpretation of Marshall as presented in an earlier and much briefer draft of this article.

[2] C. W. Guillebaud, "The Evolution of Marshall's *Principles of Economics*," *Economic Journal*, LII (December, 1942), 333.

ical inconsistency and mathematical error at the very foundation of his theory of demand. More important, the alternative interpretation of the demand curve that is yielded by a literal reading of his remarks not only leaves his original work on the theory of demand free from both logical inconsistency and mathematical error but also is more useful for the analysis of most economic problems.

Section I presents the two interpretations of the demand curve and compares them in some detail; Section II argues that a demand curve constructed on my interpretation is the more useful for the analysis of practical problems, whatever may be the verdict about its validity as an interpretation of Marshall; Section III demonstrates that my interpretation is consistent with Marshall's monetary theory and with his work on consumer's surplus; and Section IV presents the textual evidence on the validity of my interpretation. Finally, Section V argues that the change that has occurred in the interpretation of the demand curve reflects a corresponding change in the role assigned to economic theory.

I. ALTERNATIVE INTERPRETATIONS OF MARSHALL'S DEMAND CURVE

The demand curve of a particular group (which may, as a special case, consist of a single individual) for a particular commodity shows the quantity (strictly speaking, the maximum quantity) of the commodity that will be purchased by the group per unit of time at each price. So far, no question arises; this part of the definition is explicit in Marshall and is common to both alternatives to be discussed. The problem of interpretation relates to the phrase, "other things the same," ordinarily attached to this definition.

In the first place, it should be noted that "same" in this phrase does not mean "same over time." The points on a demand curve are alternative possibilities, not temporally ordered combinations of quantity and price. "Same" means "same for all points on the demand curve"; the different points are to differ in quantity and price and are not to differ with respect to "other things."[3] In the second place, "all" other things cannot be supposed to be the same without completely emasculating the concept. For example, if (*a*) total money expenditure on all commodities, (*b*) the price of every commodity other than the one in question, and (*c*) the quantity purchased of every other commodity were supposed to be the same, the amount of money spent on the commodity in question would necessarily be the same at all prices, simply as a matter of arithmetic, and the demand curve would have unit elasticity everywhere.[4] Different specifications of the "other things" will yield different demand curves. For example,

[3] Of course, when correlations among statistical time series are regarded as estimates of demand curves, the hypothesis is that "other things" have been approximately constant over time or that appropriate allowance has been made for changes in them. Similarly, when correlations among cross-section data are regarded as estimates of demand curves, the hypothesis is that "other things" are approximately the same for the units distinguished or that appropriate allowance has been made for differences among them. In both cases the problem of estimation should be clearly distinguished from the theoretical construct to be estimated.

[4] Yet Sidney Weintraub not only suggests that Marshall intended to keep *a*, *b*, and *c* simultaneously the same but goes on to say: "Clearly Marshall's assumption means a unit elasticity of demand in the market reviewed and no ramifications elsewhere; that was why he adopted it" ("The Foundations of the Demand Curve," *American Economic Review*, XXXII [September, 1942], 538–52, quotation from n. 12, p. 541). Weintraub even adds the condition of constant tastes and preferences to *a*, *b*, and *c*, speaking of a change in tastes as shifting the demand curve. Obviously, *a*, *b*, and *c* together leave no room for tastes and preferences or, indeed, for anything except simple arithmetic.

one demand curve will be obtained by excluding *b* from the list of "other things"; another, quite different one, by excluding *c*.

a) THE CURRENT INTERPRETATION

The current interpretation of Marshall's demand curve explicitly includes in the list of "other things" (1) tastes and preferences of the group of purchasers considered; (2) their money income, and (3) the price of every other commodity. The quantities of other commodities are explicitly considered as different at different points on the demand curve, and still other variables are ignored.[5]

On this interpretation, it is clear that, while money income is the same for different points on the demand curve, real income is not. At the lower of two prices for the commodity in question, more of some commodities can be purchased without reducing the amounts purchased of other commodities. The lower the price, therefore, the higher the real income.

[5] Explicit definition of the demand curve in this way by followers of Marshall dates back at least to 1894 (see F. Y. Edgeworth, article on "Demand Curves" in *Palgrave's Dictionary of Political Economy*, edited by Henry Higgs [rev. ed.; London: Macmillan & Co., Ltd., 1926]). Edgeworth's article apparently dates from the first edition, which was published in 1894. While Edgeworth does not explicitly attribute this interpretation to Marshall, it is clear from the context that he is talking about a Marshallian demand curve and that he does not regard his statements as inconsistent in any way with Marshall's *Principles*. Though no explicit listing of "other things" is given by J. R. Hicks, *Value and Capital* (Oxford, 1939), the list given above is implicit throughout chaps. i and ii, which are explicitly devoted to elaborating and extending Marshall's analysis of demand. For statements in modern textbooks on advanced economic theory see G. J. Stigler, *The Theory of Price* (New York: Macmillan Co., 1946), pp. 86–90, and Kenneth E. Boulding, *Economic Analysis* (rev. ed., New York: Harper & Bros., 1948), pp. 134–35.

b) AN ALTERNATIVE INTERPRETATION

It seems to me more faithful to both the letter and the spirit of Marshall's writings to include in the list of "other things" (1) tastes and preferences of the group of purchasers considered, (2) their real income, and (3) the price of every closely related commodity.

Two variants of this interpretation can be distinguished, according to the device adopted for keeping real income the same at different points on the demand curve. One variant, which Marshall employed in the text of the *Principles*, is obtained by replacing "(2) their real income" by (*2a*) their money income and (*2b*) the "purchasing power of money." Constancy of the "purchasing power of money" for different prices of the commodity in question implies compensating variations in the prices of some or all other commodities. These variations will, indeed, be negligible if the commodity in question accounts for a negligible fraction of total expenditures; but they should not be disregarded, both because empirical considerations must be sharply separated from logical considerations and because the demand curve need not be limited in applicability to such commodities. On this variant, all commodities are, in effect, divided into three groups: (*a*) the commodity in question, (*b*) closely related commodities, and (*c*) all other commodities. The absolute price of each commodity in group *b* is supposed to be the same for different points on the demand curve; only the "average" price, or an index number of prices, is considered for group *c;* and it is to be supposed to rise or fall with a fall or rise in the price of group *a*, so as to keep the "purchasing power of money" the same.

The other variant, which Marshall employed in the Mathematical Appendix of

the *Principles*, is obtained by retaining "(2) their real income" and adding (4) the average price of all other commodities. Constancy of real income for different prices of the commodity in question then implies compensating variations in money income. As the price of the commodity in question rises or falls, money income is to be supposed to rise or fall so as to keep real income the same.

These two variants are essentially equivalent mathematically,[6] but the assumption of compensating variations in other prices is easier to explain verbally and can be justified as empirically relevant by considerations of monetary theory, which is presumably why Marshall used this variant in his text. On the other hand, the assumption of compensating variations in income is somewhat more convenient mathematically, which is presumably why Marshall used this variant in his Mathematical Appendix.

On my interpretation, Marshall's demand curve is identical with one of the constructions introduced by Slutsky in his famous paper on the theory of choice, namely, the reaction of quantity demanded to a "compensated variation of price," i.e., to a variation in price accompanied by a compensating change in money income.[7] Slutsky expressed the compensating change in money income in terms of observable phenomena, taking it as equal to the change in price *times* the quantity demanded at the initial price. Mosak has shown that, in the

[6] Let x and y be the quantity and price, respectively, of the commodity in question; x' and y' the quantity and price of a composite commodity representing all other commodities; and m, money income. Let

$$x = g(y, y', m, u) \tag{1}$$

be the demand curve for the commodity in question, given a utility function,

$$U = U(x, x', u), \tag{2}$$

where u is a parameter to allow for changes in taste, and subject to the condition

$$xy + x'y' = m. \tag{3}$$

From eq. (3) and the usual utility analysis, it follows that eq. (1), like eq. (3), is a homogeneous function of degree zero in y, y', and m; i.e., that

$$g(\lambda y, \lambda y', \lambda m, u) = g(y, y', m, u). \tag{4}$$

On the current interpretation, a two-dimensional demand curve is obtained from eq. (1) directly by giving y' (other prices), m (income), and u (tastes) fixed values. A given value of y then implies a given value of x from eq. (1), a given value of x' from eq. (3), and hence a given value of U (i.e., real income) from eq. (2). The value of U will vary with y, being higher, the lower y is.

On my alternative interpretation, u and U are given fixed values and x' is eliminated from eqs. (2) and (3). This gives a pair of equations,

$$x = g(y, y', m, u_0), \tag{5}$$

$$U_0 = U_0\left(x, \frac{m - xy}{y'}, u_0\right), \tag{6}$$

where the subscript o designates fixed values. The two-dimensional variant involving compensating variations in other prices is obtained by eliminating y' from eqs. (5) and (6) and giving m a fixed value; the variant involving compensating variations in income, by eliminating m from eqs. (5) and (6) and giving y' a fixed value.

The homogeneity of eqs. (5) and (6) in y, y', and m means that x is a function only of ratios among them. Thus eqs. (5) and (6) can be written:

$$\left.\begin{aligned} x = g(y, y', m, u_0) &= g\left(\frac{y}{m}, \frac{y'}{m}, 1, u_0\right) \\ &= g\left(\frac{y}{y'}, 1, \frac{m}{y'}, u_0\right), \end{aligned}\right\} \tag{5'}$$

$$\left.\begin{aligned} U_0 = U_0\left(x, \frac{m - xy}{y'}, u_0\right) &= U_0\left(x, \frac{1 - x\frac{y}{m}}{\frac{y'}{m}}, u_0\right) \\ &= U_0\left(x, \frac{m}{y'} - x\frac{y}{y'}, u_0\right). \end{aligned}\right\} \tag{6'}$$

The choice of price-compensating variations is equivalent to selecting the forms of these two equations in the next to the last terms of eqs. (5') and (6'); of income-compensating variations, to selecting the forms in the last terms.

[7] Eugenio Slutsky, "Sulla teoria del bilancio del consumatore," *Giornale degli economisti*, LI (1915), 1–26, esp. Sec. 8.

limit, the change in income so computed is identical with the change required to keep the individual on the same level of utility (on the same indifference curve).[8] It follows that a similar statement is valid for compensating changes in other prices. In the limit, the change in other prices required to keep the individual on the same indifference curve when his money income is unchanged but the price of one commodity varies is identical with the change in other prices required to keep unchanged the total cost of the basket of commodities purchased at the initial prices, i.e., to keep unchanged the usual type of cost-of-living index number.

c) COMPARISON OF THE INTERPRETATIONS

The relation between demand curves constructed under the two interpretations is depicted in Figure 1. Curve *Cc* represents a demand curve of an individual consumer for a commodity *X* drawn on the current interpretation. Money income and the prices of other commodities are supposed the same for all points on it; in consequence, real income is lower at *C* than at *P*, since, if the individual sought to buy *OM* of *X* at a price of *OC*, he would be forced to curtail his purchases of something else. As the curve is drawn, of course, he buys none of *X* at a price of *OC*, spending the sum of *OHPM* on other commodities that his action at a price of *OH* shows him to value less highly than he does *OM* units of *X*. The ordinate is described as the ratio of the price of *X* to the price of other commodities. For the demand curve *Cc* this is a question only of the unit of measure, since other prices are supposed to be the same for all points on it.

From the definition of the demand curve *Cc*, *OC* is obviously the maximum price per unit that an individual would be willing to pay for an infinitesimal initial increment of *X* when his money income and the prices of other commodities have the values assumed in drawing *Cc*. Let us suppose him to purchase this amount at a price of *OC*, determine the

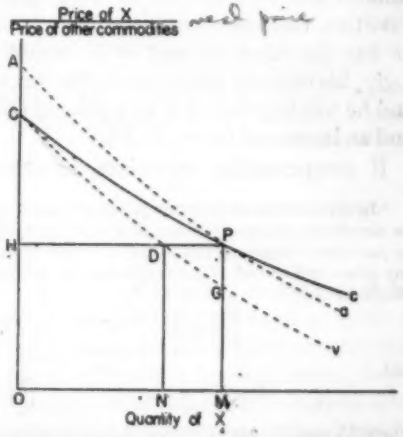
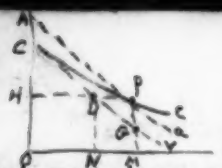


FIG. 1.—Comparison of demand curves constructed under the two interpretations.

maximum price per unit he would be willing to pay for an additional increment, and continue in this fashion, exacting the maximum possible amount for each additional increment. Let these successive maximum prices per unit define the curve *Cv*. The consumer obviously has the same real income at each point on *Cv* as at *C*, since the maximum price has been extracted from him for each successive unit, so that he has gained no utility in the process.

Cv is now a demand curve constructed according to my interpretation of Marshall. If other prices are supposed to be the same, the necessary compensating

[8] Jacob L. Mosak, "On the Interpretation of the Fundamental Equation of Value Theory," in O. Lange, F. McIntyre, and T. O. Yntema, *Studies in Mathematical Economics and Econometrics* (Chicago: University of Chicago Press, 1942), pp. 69–74, esp. n. 5, pp. 73–74, which contains a rigorous proof of this statement by A. Wald.



variations in money income as the price of X falls are given by triangular areas exemplified by HCD for a price of OH: OH is the maximum price per unit that the individual will give for an additional infinitesimal increment of X when he has spent $OCDN$ for ON of X out of his initial income of, say, m; but his situation is exactly the same if, when the price of X is OH, his income is $(m - HCD)$ and he spends $OHDN$ on X; he has the same amount left to spend on all other commodities, their prices are the same, and he has the same amount of X; accordingly, his demand price will be the same, and he will buy ON of X at a price of OH and an income of $(m - HCD)$.[9]

If compensating variations in other prices rather than in money income are used to keep real income the same, the absolute price of neither X nor other commodities can be read directly from Figure 1. For each ratio of the price of X to the price of other commodities, the quantity of X purchased will be that shown on Cv. But the prices of other goods will vary along Cv, rising as the relative price of X falls, so the absolute price of X can no longer be obtained by multiplying the ordinate by a single scale factor.

Figure 1 is drawn on the assumption that X is a "normal" commodity, that is, a commodity the consumption of which is higher, the higher the income. This is the reason Cv is drawn to the left of Cc—at every point on Cv other than C, real income is less than at the corresponding point on Cc; hence less X would be consumed.

Curve Aa represents a demand curve on my interpretation of Marshall for a real income the same as at point P on Cc; it is like Cv but for a higher real income. Real income is higher on Aa than on Cc for prices above OH, lower for prices below OH, which is the reason Aa is to the right of Cc for prices above OH and to the left of Cc for prices below OH.

d) WHY TWO INTERPRETATIONS ARE POSSIBLE

The possibility of interpreting Marshall in these two quite different ways arises in part from the vagueness of Marshall's exposition, from his failure to give precise and rigorous definitions. A more fundamental reason, however, is the existence of inconsistency in the third and later editions of the *Principles*. In that edition Marshall introduced the celebrated passage bearing on the Giffen phenomenon. This passage and a related sentence added at the same time to the

[9] In the notation of n. 6, except that u is omitted for simplicity, the quantities of X and X' that will be purchased for any given values of y and y' and any given real income, U_0, are obtained by solving simultaneously:

$$\frac{U_x}{U_{x'}} = \frac{y}{y'}, \tag{1}$$

and

$$U(x, x') = U_0, \tag{2}$$

where U_x and $U_{x'}$ stand for the partial derivatives of U with respect to x and x', respectively, i.e., for the marginal utility of X and X'. The solution of these equations gives the demand curve on my interpretation of Marshall, using compensating variations in money income.

$U_0(o, m/y')$ is the utility at C in the diagram. For any given amount of X and given value of y', the amount of X' purchased is obtained by solving

$$U(x, x') = U_0\left(o, \frac{m}{y'}\right), \tag{3}$$

which is identical with eq. (2). The amount paid for X (the area under Cv) is

$$m - x'y'. \tag{4}$$

The maximum price that will be paid per unit of X is the derivative of eq. (4), or

$$y = -\frac{dx'}{dx}y' = \frac{U_x}{U_{x'}}y', \tag{5}$$

which is identical with eq. (1). It follows that Cv is a demand curve constructed on my interpretation of Marshall.

Mathematical Appendix fit the current interpretation better than they fit my interpretation. Although these are the only two items that I have been able to find in any edition of the *Principles* of which this is true, they provide some basis for the current interpretation. A hypothesis to explain the introduction of this inconsistency into the *Principles* is offered in Section IV*e* below.

II. THE RELATIVE USEFULNESS OF THE TWO INTERPRETATIONS

The relative usefulness of the two interpretations of the demand curve can be evaluated only in terms of some general conception of the role of economic theory. I shall use the conception that underlies Marshall's work, in which the primary emphasis is on positive economic analysis, on the forging of tools that can be used fairly directly in analyzing practical problems. Economic theory was to him an "engine for the discovery of concrete truth."[10] "Man's powers are limited: almost every one of nature's riddles is complex. He breaks it up, studies one bit at a time, and at last combines his partial solutions with a supreme effort of his whole small strength into some sort of an attempt at a solution of the whole riddle."[11] The underlying justification for the central role of the concepts of demand and supply in Marshall's entire structure of analysis is the empirical generalization that an enumeration of the forces affecting demand in any problem and of the forces affecting supply will yield two lists that contain few items in common. Demand and supply are to him concepts for organizing materials, labels in an "analytical filing box." The "commodity" for which a demand curve is drawn is another label, not a word for a physical or technical entity to be defined once and for all independently of the problem at hand. Marshall writes:

> The question where the lines of division between different commodities should be drawn must be settled by convenience of the particular discussion. For some purposes it may be best to regard Chinese and Indian teas, or even Souchong and Pekoe teas, as different commodities; and to have a separate demand schedule for each of them. While for other purposes it may be best to group together commodities as distinct as beef and mutton, or even as tea and coffee, and to have a single list to represent the demand for the two combined.[12]

a) THE DISTINCTION BETWEEN CLOSELY RELATED AND ALL OTHER COMMODITIES

A demand function containing as separate variables the prices of a rigidly defined and exhaustive list of commodities, all on the same footing, seems largely foreign to this approach. It may be a useful expository device to bring home the mutual interdependence of economic phenomena; it cannot form part of Marshall's "engine for the discovery of concrete truth." The analyst who attacks a concrete problem can take explicit account of only a limited number of factors; he will inevitably separate commodities that are closely related to the one immediately under study from commodities that are more distantly related. He can pay some attention to each closely related commodity. He cannot handle the more distantly related commodities in this way; he will tend either to ignore

[10] Alfred Marshall, "The Present Position of Economics" (1885), reprinted in *Memorials of Alfred Marshall*, ed. A. C. Pigou (London: Macmillan & Co., Ltd., 1925), p. 159.

[11] Alfred Marshall, "Mechanical and Biological Analogies in Economics" (1898), *ibid.*, p. 314.

[12] Marshall, *Principles of Economics* (8th ed.; London: Macmillan & Co., Ltd., 1920), p. 100 n. All subsequent page references to the *Principles*, unless otherwise stated, are to the eighth and final edition.

them or to consider them as a group. The formally more general demand curve will, in actual use, become the kind of demand curve that is yielded by my interpretation of Marshall.

The part of the Marshallian filing box covered by *ceteris paribus* typically includes three quite different kinds of variables, distinguished by their relation to the variable whose adaptation to some change is directly under investigation (e.g., the price of a commodity): (*a*) variables that are expected both to be materially affected by the variable under study and, in turn, to affect it; (*b*) variables that are expected to be little, if at all, affected by the variable under study but to materially affect it; (*c*) the remaining variables, expected neither to affect significantly the variable under study nor to be significantly affected by it.

In demand analysis the prices of closely related commodities are the variables in group *a*. They are put individually into the pound of *ceteris paribus* to pave the way for further analysis. Holding their prices constant is a provisional step. They must inevitably be affected by anything that affects the commodity in question; and this indirect effect can be analyzed most conveniently by first isolating the direct effect, systematically tracing the repercussions of the direct effect on each closely related commodity, and then tracing the subsequent reflex influences on the commodity in question. Indeed, in many ways, the role of the demand curve itself is as much to provide an orderly means of analyzing these indirect effects as to isolate the direct effect on the commodity in question.

The average price of "all other commodities," income and wealth, and tastes and preferences are the variables in group *b*. These variables are likely to be affected only negligibly by factors affecting primarily the commodity in question. On the other hand, any changes in them would have a significant effect on that commodity. They are put into the pound in order to separate problems, to segregate the particular reactions under study. They are put in individually and explicitly because they are so important that account will have to be taken of them in any application of the analysis.

Price changes within the group of "all other commodities" and an indefinitely long list of other variables are contained in group *c*. These variables are to be ignored. They are too numerous and each too unimportant to permit separate account to be taken of them.

In keeping with the spirit of Marshallian analysis this classification of variables is to be taken as illustrative, not definitive. What particular variables are appropriate for each group is to be determined by the problem in hand, the amount of information available, the detail required in results, and the patience and resources of the analyst.

b) CONSTANCY OF REAL INCOME

It has just been argued that any actual analysis of a concrete economic problem with the aid of demand curves will inevitably adopt one feature of my interpretation of Marshall—consideration of a residual list of commodities as a single group. For somewhat subtler reasons this is likely to be true also of the second feature of my interpretation of Marshall—holding real income constant along a demand curve. If an analysis, begun with a demand curve constructed on the current interpretation, is carried through and made internally consistent, it will be found that the demand curve has been subjected to shifts that, in effect, result from failure to keep real income constant along the demand curve.

An example will show how this occurs. Let us suppose that the government grants to producers of commodity X a subsidy of a fixed amount per unit of output, financed by a general income tax, so that money income available for expenditure (i.e., net of tax and gross of subsidy) is unchanged. For simplicity, suppose, first, that no commodities are closely related to X either as rivals or as complements, so that interrelations in consumption between X and particular other commodities can be neglected; second, that the tax is paid by individuals in about the same income class and with about the same consumption pattern as those who benefit from the subsidy, so that complications arising from changes in the distribution of income can be neglected; and, third, that there are no idle resources. Let DD in Figure 2 be a demand curve for commodity X, and SS be the initial supply curve for X, and let the initial position at their intersection, point P, be a position of full equilibrium. The effect of the subsidy is to lower the supply curve to $S'S'$. Since we have ruled out repercussions through consumption relations with other markets and through changes in the level or distribution of money income, it is reasonable to expect that the intersection of this new supply curve and the initial demand curve, point P', will itself be a position of full equilibrium, involving a lower price and larger quantity of X. Yet, if the demand curve is constructed on the current interpretation and if the supply curve is not perfectly inelastic,[13] point P' is not a position of full equilibrium. This can be seen most easily by supposing DD to have unit elasticity, so that the same amount is spent on X at P' as at P. The same amount is then available to spend on all other commodities, and, since their prices are supposed to be the same for all points on DD under the current interpretation, the same quantity of each of them will be demanded. But then where do the resources come from to produce the extra MN units of X? Obviously, our assumptions are not internally consistent. The additional units of X can be produced only by bidding resources away from the production of other commodities, in the process raising their prices and reducing the amount of them produced. The final equilibrium position will therefore involve higher prices and lower quantities of other commodities. But, on the current interpretation, this means a shift in the demand curve for X—say, to $D'D'$—and a final equilibrium position of, say, P''.[14]

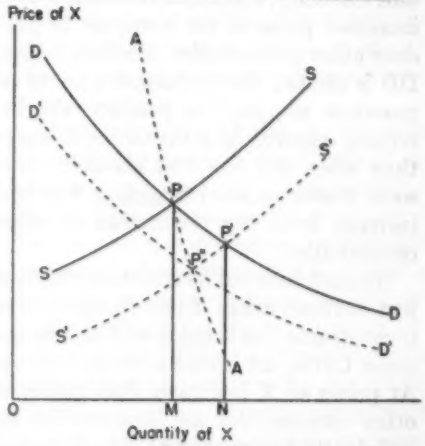
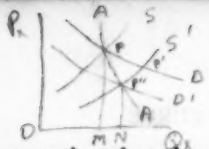


FIG. 2.—Illustrative analysis of effect of subsidy

[13] If it is perfectly inelastic, neither the price nor the quantity of X is changed, so the new position of equilibrium coincides with the old; but the demand curve will pass through the initial position of equilibrium whether constructed on the current interpretation or on mine; hence the two coincide at the one point on them that is relevant.

[14] $D'D'$ will not necessarily be to the left of DD even for a "normal" commodity. The reason is that the ordinate of Fig. 2 measures the absolute price of X, so that ordinates of the same height on DD



The assumption that the elasticity of *DD* is unity is not, of course, essential for this argument. If the elasticity of *DD* is less than unity, a larger amount than formerly is available to spend on other commodities; at unchanged prices this means a larger quantity demanded. In consequence, while the additional amount of resources required to produce the increased amount of *X* demanded is smaller when *DD* is inelastic than when it has unit elasticity, this is counterbalanced by increased pressure for resources to produce other commodities. Similarly, when *DD* is elastic, the additional amount of resources required to produce the increased quantity of *X* demanded is larger than when *DD* has unit elasticity, but some resources are released in the first instance from the production of other commodities.

No such internal inconsistency as that just outlined arises if the demand curve is constructed by keeping real income the same. Curve *AA* is such a demand curve. At prices of *X* less than *PM*, prices of other commodities are supposed to be sufficiently higher than at *P* to keep real income the same, which involves the release of just enough resources so that the position of final equilibrium, *P''*, lies on the demand curve so constructed—at least for small changes in the price of *X*.[15]

The fundamental principle illustrated by this example can be put more generally. The reason why a demand curve constructed under the current interpretation fails to give the correct solution even when all disturbing influences can be neglected is that each point on it implicitly refers to a different productive capacity of the community. A reduction in the price of the commodity in question is to be regarded as enabling the community, if it so wishes, to consume more of some commodities—this commodity or others—without consuming less of any commodity. But the particular change in supply whose consequences we sought to analyze—that arising from a subsidy—does not make available any additional resources to the community; any increase in the consumption of the commodity in question must be at the expense of other commodities. The conditions for which the demand curve is drawn are therefore

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and *D'D'* represent different ratios of the price of *X* to the price of other commodities. If the ordinate measured the ratio of the price of *X* to the price of other commodities, *D'D'* would always be to the left of *DD* for "normal" commodities, always to the right for "inferior" commodities.

[15] Let *X'* be a single composite commodity representing all commodities other than *X*; *x* and *x'*, the quantities of *X* and *X'*; and *y* and *y'*, their prices. Let the subscript 1 refer to values at the initial position of equilibrium, *P*; the subscript 2, to values at the final position, *P''*. The condition of constant total expenditures means that

$$x_1y_1 + x'_1y'_1 = x_2y_2 + x'_2y'_2 . \qquad (1)$$

As was pointed out above (Sec. Ib), in the limit, holding real income constant is equivalent to holding constant the cost of a fixed basket of commodities. Thus, if *P''* is considered close to *P*,

$$x_1y_1 + x'_1y'_1 = x_1y_2 + x'_1y'_2 . \qquad (2)$$

In the neighborhood of *P*, y_1 can be regarded as the cost per unit of producing *X*; y'_1, as the cost per unit of producing *X'*. The condition that sufficient resources be released to permit the production of the requisite additional amount of *X* is therefore

$$(x_2 - x_1)y_1 = -(x'_2 - x'_1)y'_1 , \qquad (3)$$

which is equivalent to

$$x_1y_1 + x'_1y'_1 = x_2y_1 + x'_2y'_1 . \qquad (4)$$

But, in the limit, eqs. (1) and (2) imply eq. (4), as can be seen by subtracting eq. (2) from eq. (1) and replacing y_2 and y'_2 in the result by $(y_2 - y_1 + y_1)$ and $(y'_2 - y'_1 + y'_1)$, respectively.

More generally, constant real income involves keeping a price index unchanged; constant use of resources involves keeping a quantity index unchanged; and, in the limit, a constant price index and constant total expenditures imply a constant quantity index.

Note that *AA* need not be steeper than *DD* in a graph like Fig. 2. The point in question is that commented on in n. 14.

inconsistent with the conditions postulated on the side of supply. On the other hand, if the demand curve is constructed by keeping "real income" the same, no such inconsistency need arise. True, constant "real income" in the sense of "utility" and constant "real income" in the sense of outputs attainable from a fixed total of resources are different concepts, but they converge and can be treated as the same in the neighborhood of a position of equilibrium.

Of course, not all shifts in supply that it is desired to analyze arise in ways that leave the productive capacity of the community unaltered. Many involve a change in productive capacity—for example, changes in supply arising from improvements in technology or the discovery of previously unknown resources. Even in these cases, however, a demand curve constructed on the current interpretation will not serve. There is no reason to expect the differences in productive capacity implicit in constant money income and constant prices of other goods to bear any consistent relation to the change in productive capacity arising on the side of supply.[16] The better plan, in these cases, is to allow separately and directly for the increase in productive capacity by redrawing the demand curves to correspond to an appropriately higher real income and then to use a demand curve on which all points refer to that higher real income.

The main point under discussion can be put still more generally. The opportunities open to a consumer to satisfy his wants depend principally on two factors—the total resources at his disposal and the terms on which he can exchange one commodity for another, that is, on his real income and on relative prices. The form of analysis that is now fashionable distinguishes three effects of changes in his opportunities—the income effect arising from changes in his money income; the income effect arising from changes in the price of a commodity, with unchanged money income and prices of other commodities; and the substitution effect arising from a change in the relative price of a commodity, with unchanged real income.

The distinction between the so-called "substitution" and "income" effects of a change in price is a direct consequence of defining the demand curve according to the current interpretation of Marshall. Its basis is the arithmetic truism that at given prices for all commodities but one, a given money income corresponds to a higher real income, the lower the price of the remaining commodity—at a lower price for it, more of some commodities can be purchased without purchasing less of others. In consequence, a decline in the price of a commodity, all other prices constant, has, it is argued, two effects: first, with an unchanged real income, it would stimulate the substitution of that commodity for others—this is the substitution effect; second, if the money income of the consumers is supposed to be unchanged, the increase in their real income as a result of the decline in price causes a further change in the consumption of that commodity as well as of others—this is the income effect.[17]

[16] Note the difference from the previous case of constant productive capacity. As stated above, there is reason to expect constant real income along a demand curve to bear a consistent relation to constant productive capacity in the neighborhood of equilibrium. The reason, in effect, is provided by one of the conditions of equilibrium: the tangency of consumption and production indifference curves,

[17] See Slutsky, *op. cit.*; Henry Schultz, *The Theory and Measurement of Demand* (Chicago: University of Chicago Press, 1938), pp. 40–46; J. R. Hicks and R. G. D. Allen, "A Reconsideration of the Theory of Value," *Economica*, XIV (1934), 52–76 and 196–219; Hicks, *op. cit.*, Part I.

The two different kinds of income effects distinguished in this analysis—one arising from a change in money income, the other from a change in the price of one commodity—are really the same thing, the effect of a change in real income with given relative prices, arising in different ways. It is hard to see any gain from combining the second income effect with the substitution effect; it seems preferable to combine the two income effects and thereby gain a sharp contrast with the substitution effect.

It has often been stated that Marshall "neglected the income effect."[18] On my interpretation of his demand curve, this statement is invalid. One must then say that Marshall recognized the desirability of separating two quite different effects and constructed his demand curve so that it encompassed solely the effect that he wished to isolate for study, namely, the substitution effect. Instead of neglecting the income effect, he "eliminated" it.

The conclusion to which the argument of this section leads is identical with that reached by Frank H. Knight in a recent article, in which he says:

> We have to choose in analysis between holding the prices of all other goods constant and maintaining constant the "real income" of the hypothetical consumer. . . . The treatment of the Slutzky school adopts the assumption that . . . the prices of all other goods (and the consumer's money income) are constant. Hence, real income must change. Of the two alternatives, this seems to be definitely the wrong choice. . . . The simple and obvious alternative is to draw the demand curves in terms of a change in *relative* prices, i.e., to assume that the value of money is held constant, through compensating changes in the prices of other goods, and not that these other prices are held constant.[19]

[18] Hicks, *op. cit.*, p. 32.

[19] "Realism and Relevance in the Theory of Demand," *Journal of Political Economy*, LII (December, 1944), 289–318, esp. Sec. III, "The Meaning of a Demand Curve," pp. 298–301. Quotation from p. 299.

III. THE CONSISTENCY OF THE ALTERNATIVE INTERPRETATION WITH OTHER PARTS OF MARSHALL'S WORK

Marshall's demand curve is part of a coherent body of thought; it is designed to fit into the rest of his structure of analysis; and it is used extensively in developing and applying this structure. It would take us too far afield to demonstrate in detail that my interpretation of his demand curve is consistent with the rest of his work. However, two special topics call for some explicit consideration: (1) the relation between the demand curve and Marshall's theory of money, because, in my view, this explains the particular device that he adopted for holding real income constant; and (2) the concept of consumer's surplus, because this is one of the most important applications of the demand curve and certainly the most controversial and because the passages in the later editions of the *Principles* that are inconsistent with my interpretation were introduced into the discussion of consumer's surplus.

a) THE THEORY OF RELATIVE PRICES AND THE THEORY OF MONEY

Granted that real income is to be held constant along the demand curve, why do so by holding money income and the purchasing power of money constant rather than, for example, by holding prices of other goods constant and permitting compensating variations in money income? What reason is there to treat the prices of all other commodities as moving inversely to the price of the commodity in question?

The answer to these questions is given, I think, by one of Marshall's basic or-

ganizing principles, namely, the separation of the theory of relative prices from monetary theory, the theory of the level of prices. The *Principles* is devoted to the theory of relative prices under given monetary conditions; *Money, Credit, and Commerce* to the analysis of monetary conditions and their effect on the "purchasing power of money." With *given monetary conditions*, is it possible for the prices of all commodities other than the one in question to remain the same, on the average, while the price of this one rises or falls? Will not a rise or fall in the price of the commodity in question set in motion *monetary* forces affecting other prices? A complete answer requires explicit specification of the content of "given monetary conditions" and perhaps also of the source of the initial price change.

Marshall's selection of a constant purchasing power of money as a means of impounding monetary forces is presumably the end-result of a chain of reasoning about the influence of monetary forces, not the direct content that he gave to "given monetary conditions." The beginning of the chain of reasoning may well be his own version of the quantity theory of money. According to this version, "the value of money is a function of its supply on the one hand, and the demand for it, on the other, as measured by 'the average stock of command over commodities which each person cares to keep in a ready form.' "[20] Given monetary conditions would then imply a given stock of money and a given desired "average stock of command over commodities." A decline in one price alone, all other prices remaining the same, is inconsistent with these "givens." It would increase the real value of a fixed (nominal) stock of money, leave the community with a larger "stock of command over commodities" than previously, and establish an incentive (reflecting "monetary" forces) to increase expenditures and thereby raise prices until the fixed stock of money again represented the same "stock of command over commodities," i.e., until the "purchasing power of money" reached its former level.[21] This argument suggests that not only was constant purchasing power of money a device for separating the theory of relative prices from monetary theory, it was also a bridge between the two. Marshall separated the two theories in his attempt to reduce problems to manageable proportions, but he constructed them in such a way as to make them mutually consistent and thus facilitate ultimate combination.[22]

[20] J. M. Keynes, "Alfred Marshall, 1842–1924," *Memorials*, p. 297.

[21] C. W. Guillebaud has pointed out to me that Marshall typically supposed the desired "stock of command over commodities" to be a given fraction of real income (see *ibid.*) and that the argument in the text might not apply if this fraction were taken as the fundamental given. The monetary effects of a change in one price, other prices given, would then depend on the source of the initial price change. If this involved no change in aggregate real income (e.g., arose from a shift in demand), the argument in the text would remain unchanged. If it did involve a change in aggregate real income (e.g., arose from an invention reducing the cost of producing the commodity in question), no inconsistency need arise, since the desired "stock of command over commodities" would change in proportion to the change in real income. These considerations account for the phrase "perhaps also of the source of the initial price change" at the end of the preceding paragraph of the text.

[22] This interpretation would, of course, be contradicted if Marshall had devised his theory of money after he had substantially completed his theory of relative prices, as might be inferred from the fact that *Money, Credit, and Commerce* was not published until 1923, thirty-three years after the first edition of the *Principles*. But in Marshall's case, the order of publication is a poor guide to the order of construction. Keynes tells us that the essence of his quantity theory of money is contained in a manuscript "written about 1871"; that "by 1871 his progress along" the lines of the material

Marshall was, of course, very much aware of the interaction between real and monetary factors. The 1879 *Economics of Industry* contains an extremely interesting discussion of the trade cycle, part of which Marshall thought sufficiently important to quote at length in 1886 in answering questions circulated by the celebrated Royal Commission on the Depression of Trade and Industry.[23]

Marshall's decision to keep the purchasing power of money the same for different points on a demand curve may not be the device best suited to abstract from monetary factors. It serves, however, to emphasize the necessity of considering explicitly the monetary arrangements under which the forces affecting relative prices are supposed to operate. The best apparatus for tackling problems of relative prices cannot be determined independently of these arrangements and of their mode of operation. Though price theory and monetary theory can be separated, they are not basically independent. From this point of view it is entirely natural that the recent development of alternative monetary theories should have stimulated re-examination of price theory.

b) CONSUMER'S SURPLUS

Marshall's discussion of consumer's surplus constitutes one of the most extensive applications that he made of his demand curve and has probably given rise to more controversy and discussion than any other part of his theory. Recently, consumer's surplus has come in for renewed attention, primarily as a result of J. R. Hick's attempt to rehabilitate and reinterpret the concept.[24] The reason for commenting on it here is not to contribute to the discussion or to evaluate the merits or demerits of the concept but rather to show the relation between Marshall's treatment of consumer's surplus and my interpretation of his demand curve.

Marshall's treatment of consumer's surplus might, offhand, seem inconsistent with my interpretation of his demand curve for either of two different, and almost opposed, reasons. In the first place, consumer's surplus refers to a difference in real income under different situations. But, on my interpretation, all

contained in the *Pure Theory* "was considerably advanced"; that the *Pure Theory* itself was "substantially complete about 1873," though not printed even for private circulation until 1879; that "in 1877 he turned aside to write the *Economics of Industry* with Mrs. Marshall"; and that Mrs. Marshall said "Book III on Demand was largely thought out and written on the roof at Palermo, Nov. 1881—Feb. 1882" (*Memorials*, pp. 28, 21, 23, 39 n.). These dates are extremely suggestive, particularly since the constancy of the purchasing power of money is not explicitly mentioned in the *Pure Theory*, which Marshall was presumably working on at about the same time that he was developing his monetary theory, while it is explicitly mentioned in the 1879 *Economics of Industry*, begun some years later. See also nn. 37 and 38 below.

[23] See Alfred Marshall and Mary Paley Marshall, *Economics of Industry* (London: Macmillan & Co., 1st ed., 1879; 2d ed., 1881), Book III, chap. i, pp. 150–57. This and all later references are to the first edition. "Answers to Questions on the Subject of Currency and Prices Circulated by the Royal Commission on the Depression of Trade and Industry (1886)," *Official Papers by Alfred Marshall* (London: Macmillan & Co., Ltd., 1926), pp. 7–9. See also "Remedies for Fluctuations of General Prices" (1887), *Memorials*, pp. 189–92.

[24] See Hicks, *op. cit.*, pp. 38–41; "The Rehabilitation of Consumers' Surplus," *Review of Economic Studies*, VIII (February, 1941), 108–16; "Consumers' Surplus and Index Numbers," *ibid.* (summer, 1942), 126–37; "The Four Consumer's Surpluses," *ibid.*, XI (winter, 1943), 31–41. See also A. Henderson, "Consumer's Surplus and the Compensating Variation," *Review of Economic Studies*, VIII (February, 1941), 117–21; Knight, *op. cit.*; Kenneth E. Boulding, "The Concept of Economic Surplus," *American Economic Review*, XXXV (December, 1945), 851–69, reprinted in American Economic Association, *Readings in the Theory of Income Distribution* (Philadelphia: Blakiston Co., 1946), pp. 638–59; E. J. Mishan, "Realism and Relevance in Consumer's Surplus," *Review of Economic Studies*, XV (1947–48), 27–33.

points on the demand curve are to be regarded as corresponding to the same real income. A movement along such a demand curve cannot, therefore, involve a change in consumer's surplus. Does this not eliminate the entire notion of consumer's surplus and make Marshall's entire discussion of it pointless? The answer is clearly "No," the reason being that the two situations compared need not correspond to two points on the *same* demand curve, even though a single demand curve is used to *estimate* the difference in real income between the two situations.

In the second place, Marshall regarded his analysis of consumer's surplus as valid only for commodities that account for a small part of total expenditure. He makes this restriction in order to justify neglecting changes in the marginal utility of money. But, if all points on the demand curve correspond to the same real income, does it not then follow that the marginal utility of money is the same everywhere on the demand curve? And does it not also follow that his estimate of consumer's surplus is exact, so that the assumption that a negligible proportion of expenditures is devoted to the commodity in question becomes unnecessary? Again the answer is "No," and for much the same reason. If the two situations compared differ in real income, the fact that real income is the same along the demand curve becomes something of a vice in using it to measure consumer's surplus. The assumption that a negligible proportion of expenditures is devoted to the commodity in question cannot be dispensed with on my interpretation; indeed, if anything, it is even more necessary than on the current interpretation.

To explain and justify these cryptic answers, it will be necessary to examine Marshall's definition of consumer's surplus, his suggested estimate of its magnitude, and the relation of this estimate to the correct value under the two alternative interpretations of the demand curve.

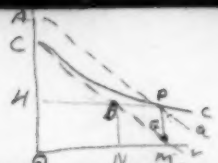
Marshall is more explicit and complete in defining consumer's surplus than was his wont, and his definition admits of little ambiguity: "The excess of the price which he would be willing to pay rather than go without the thing, over that which he actually does pay, is the economic measure of this surplus satisfaction. It may be called *consumer's surplus*."[25]

Marshall then proceeds to argue that consumer's surplus can be *estimated* by the famous triangle under the demand curve. As Hicks remarks, this "association of Consumer's Surplus with the curvilinear triangle under the demand curve . . . is not a definition; it is a theorem, true under certain restrictive assumptions, but only true if these assumptions are granted."[26] The confusion of the suggested estimate with the definition is perhaps the chief source of misunderstanding on this exceedingly complex subject.

Figure 1, introduced in Section Ic above to illustrate the relation between demand curves drawn on the current and on my interpretation, can also be used to show the relation between consumer's surplus as defined and estimates of it obtained from demand curves constructed according to the two interpretations. Curve *Cc*, it will be recalled, is a demand curve for the commodity *X* constructed according to the current interpretation. Money income and all other prices are the same for all points on it. *Aa* and *Cv*

[25] *Principles*, p. 124.

[26] "Rehabilitation of Consumers' Surplus," p. 109.



are demand curves constructed according to my interpretation—*Aa* for a real income the same as at *P* on *Cc*; *Cv* for a real income the same as at *C* on *Cc*. At point *P* on *Aa* and at point *C* on *Cv*, money income and all other prices are the same as on *Cc*. At other points other prices are sufficiently different, or money income is, to compensate for the difference in the price of *X* and thereby keep real income the same.

Now consider the consumer's surplus obtained from this commodity when the consumer is at *P*.[27] This is *defined* as "the excess of the price which he would be willing to pay rather than go without the thing, over that which he actually does pay." "Price" is here to be interpreted as "total amount" rather than "price per unit."[28] Further, it is clear that the sum he would pay rather than go without is to be determined for circumstances otherwise the same as at *P;* in particular, his money income and the other prices are to be the same as at *P*.[29] Now the amount that he actually does pay for *OM* of *X* is given by the rectangle *OHPM* in the figure. By the argument of Section I*c*, the maximum amount that he would be willing to pay for *OM* of *X* rather than go without any of it is given by the area under *Cv* between *O* and *M*, or *OCDGM*. The triangular area *CDH minus* the triangular area *DPG* therefore gives the consumer's surplus. This area is necessarily positive; we know he is willing to pay at least *OHPM* for *OM* of *X;* hence *OCDGM* must be greater than *OHPM*.

Marshall's *estimate* of the maximum sum is the area under the demand curve: *OCPM* if we use the current interpretation, *OAPM* if we use the alternative interpretation. For a "normal" commodity, the case for which the figure is drawn, both are clearly too large. How large the error is depends on the difference between *Aa* and *Cc*, on the one hand, and *Cv*, on the other. Now we have seen (in Sec. I*c*) that these differences arise entirely from differences in the real income associated with the different curves; if real incomes differ little, so will the curves. Here is where Marshall's assumption about the fraction of expenditures devoted to the commodity enters the picture. If this fraction is small, the differences in real income will tend to be small, and both estimates will approach the correct value.[30] Since the error is

[27] For simplicity, the discussion is restricted to the consumer's surplus obtained from the entire amount of *X* consumed; and to facilitate this, the demand curves have been drawn to cut the price axis.

[28] See Mathematical Note II of the *Principles* (p. 838), in which Marshall defines p as "the price which he is just willing to pay for an amount x of the commodity" and then differentiates p with respect to x to get the price per unit.

[29] None of the reasons cited earlier for keeping real income the same along the demand curve apply here. The question being asked is purely hypothetical; no other reactions need be allowed for. Further, to keep his real income the same when he has none of *X* as when he has *OM* of *X* would make the entire discussion of consumer's surplus pointless. The whole point of the discussion is to measure the difference in real income between the two situations.

[30] This statement is not rigorous. As the fraction of expenditures devoted to the commodity diminishes, so will aggregate consumer's surplus. It is not enough that the error become small in absolute terms; its ratio to the correct value must become small. This, in general, will occur, as is well known. The chief qualification has to do with the behavior of a demand curve constructed under the current interpretation (e.g., *Cc*) for small quantities of *X*. The crucial question is the difference in real income between *P* and *C*. Expenditure on the commodity might be a small fraction of total expenditure at *P;* yet, if the demand curve constructed under the current interpretation were extremely inelastic, not near *C*. In this case the difference in real income might be large.

This qualification is emphasized by Marshall. For example: "If however an amount b of the commodity is necessary for existence, $f(x)$ [*sic*] [ordinate of the demand curve] will be infinite, or at least indefinitely great, for values of x less than b. We must therefore take life for granted, and estimate separately the total utility of that part of the supply of

larger for Aa than for Cc, it is clear that Marshall's assumption is, if anything, even more necessary on my interpretation of the demand curve than on the current one.[31]

IV. TEXTUAL EVIDENCE ON WHAT MARSHALL REALLY MEANT

Marshall's writings on demand bear on three different problems: (1) the definition of the demand curve—the problem of form; (2) the shape of the demand curve—the problem of content; and (3) the use of the demand curve—the problem of application. In his usual manner Marshall gives precedence to the problem of content and does not explicitly separate his discussion of content from his discussion of form. His definitions are characteristically given parenthetically and implicitly. He went to extreme lengths to present his tools in the context of concrete problems, so that definitions grew out of the uses to be made of them.[32] His discussion of utility and diminishing utility in the chapter of the *Principles* which introduces the concept of a demand curve (Book III, chap. iii, "Gradations of Consumers' Demand") is part of the discussion of content, even though it precedes his definition. It is the means whereby he rationalizes his "one general *law of demand*:—The greater the amount to be sold, the smaller must be the price at which it is offered in order that it may find purchasers."[33] It is not part of his definition of the demand curve.

Similarly, one of the major applications that Marshall made of the demand curve was his analysis of consumer's surplus. This analysis, too, must be distinguished from his definition of the demand curve. Assumptions made in his

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the commodity which is in excess of absolute necessaries" (p. 841). See also pp. 133 and 842. $f(x)$ clearly should be $f(x)$, as it is in the first four editions of the *Principles*. See appendix to this paper.

This discussion of the role of the assumption that the commodity absorbs only a small fraction of income throws some light on an issue about which there has been considerable discussion, namely, whether Marshall assumed the marginal utility of money to be roughly constant with respect to a change in price or a change in income. The above analysis suggests that he assumed constancy with respect to a change in income. This is also Hicks's conclusion (*Value and Capital*, p. 40; "Rehabilitation," p. 109). Samuelson denies this and asserts that he assumed constancy with respect to a change in price (see Paul A. Samuelson, "Constancy of the Marginal Utility of Income," in *Studies in Mathematical Economics and Econometrics*, p. 80).

[31] The argument can be easily extended to "inferior" goods. The order of the three curves in Fig. 1 is then reversed; the estimates then become too small, instead of too large; but the error under the alternative interpretation remains larger in absolute value than under the current interpretation.

In the terminology used by Hicks in "The Four Consumers' Surpluses," what I have called the consumer's surplus is what Hicks calls the "quantity-compensating variation." The estimate of consumer's surplus derived from the demand curve constructed under my interpretation (the area APH) Hicks calls the "quantity-equivalent variation." The area CDH in Fig. 1, Hicks calls the "price-compensating variation." Hicks's fourth concept, "price-equivalent variation," is not shown directly in Fig. 1. It is obtained by drawing a horizontal line through C. Designate by E the point at which this line cuts Aa. The "price-equivalent variation" is then equal to the area APH minus AEC. These relations can be checked by noting that curve mep in Hicks's Fig. 3 is Aa in our Fig. 1; his curve PCM is Cv in our Fig. 1. Further, in comparing the two figures, the part of Hicks's diagram for quantities less than kN should be neglected. That is, his point P is equivalent to our point C, his p to our P. Our Fig. 1 is also equivalent to Fig. 3B in Boulding, "The Concept of Economic Surplus."

[32] Cf. J. M. Keynes, *Memorials*, esp. pp. 33–38; see also Guillebaud, *op. cit.*

[33] *Principles*, p. 99. Note that on my interpretation this is truly a *general* law, not subject to the exceptions that have been made in recent literature. It depends for its validity only on (*a*) the postulate that consumers can be treated as if they behaved consistently and attempted to maximize some function of the quantity of commodities consumed; (*b*) the observed fact that consumers choose a higher income in preference to a lower, other things the same; and (*c*) the observed fact that consumers do not spend all their income on one commodity. For proof that a demand curve constructed on my interpretation must slope negatively see Slutsky, *op. cit.*, Sec. 8.

discussion of consumer's surplus cannot, without additional evidence, be supposed to apply equally to other applications of the "demand curve."

a) THE CENTRAL PASSAGES IN THE TEXT OF THE "PRINCIPLES"

The central passages in the text of the eighth and final edition of the *Principles* bearing on the other things to be kept the same seem to me to be three: one governing the entire volume, and two essentially parenthetical comments in his discussion of the demand curve:

> We may throughout this volume neglect possible changes in the general purchasing power of money. Thus the price of anything will be taken as representative of its exchange value relatively to things in general [p. 62].
>
> The larger the amount of a thing that a person has the less, other things being equal (i.e. *the purchasing power of money, and the amount of money at his command being equal*), will be the price which he will pay for a little more of it: or in other words his marginal demand price for it diminishes [p. 95; italics added].
>
> The demand prices in our list are those at which various quantities of a thing can be sold in a market *during a given time and under given conditions*. If the conditions vary in any respect the prices will probably require to be changed; and this has constantly to be done when the desire for anything is materially altered by a *variation of custom, or by a cheapening of the supply of a rival commodity, or by the invention of a new one* [p. 100; second set of italics added].

For our purposes the critical part of the second quotation is the italicized parenthesis and, of the third, the second set of italicized phrases.

Though these quotations are taken from the eighth edition of the *Principles*, their substantive content is contained in Marshall's earliest published work on the theory of demand. All except the constancy of the purchasing power of money is in *The Pure Theory of (Domestic) Values*, printed for private circulation in 1879[34] but, according to Keynes, "substantially complete about 1873";[35] and the constancy of the purchasing power of money is in his and Mrs. Marshall's *The Economics of Industry*, published in 1879.[36] The actual wording of the first and third quotations can be traced back to the first edition of the *Principles* (1890), of the second quotation, to the second edition (1891).[37]

[34] Reprinted, together with the companion paper, *The Pure Theory of Foreign Trade*, by the London School of Economics and Political Science (1930).

[35] *Memorials*, p. 23.

[36] This work should not be confused with the condensation of the *Principles*, published, under the same title but with Alfred Marshall as sole author, in 1892.

[37] In all editions of the *Principles* the statement corresponding to the first quotation is in a subsection dealing with the meaning of the word "value." In the first (1890), second (1891), and third (1895) editions, the subsection on "value" is at the end of Book I, "Preliminary Survey," chap. i, "Introduction," and contains the statement: "Throughout the earlier stages of our work it will be best to speak of the exchange value of a thing at any place and time as measured by its price, that is, the amount of money for which it will exchange then and there, and to assume that there is no change in the general purchasing power of money" (p. 9, all three editions). In the first edition this assumption is repeated at the beginning of the chapter on "The Law of Demand" (Book III, chap. ii): "The purchasing power of this money may vary from time to time; but in these early stages of our work we assume it to be constant" (1st ed., p. 151). This repetition was eliminated in later editions, apparently in the process of introducing into the second edition the chapter on "Wants in Relation to Activities." In the fourth edition (1898), the subsection on "value" was split, part remaining at the end of Book I, chap. i, the remainder, including the material on the purchasing power of money, being transferred to end of Book II, "Some Fundamental Notions," chap. ii, "Wealth." The wording was changed to essentially its final form; the only difference is that the first sentence is in the passive voice, reading: "Throughout this volume possible changes in the general purchasing power of money will be neglected" (4th ed., p. 130). In the fifth edition (1907), the rest of the subsection on "value" was transferred to the end of Book II, chap. ii, and the quotation revised to its present

b) THE BEARING OF THESE PASSAGES ON THE TWO INTERPRETATIONS

The "other things" listed in the three passages cited above are as follows:

1. "Purchasing power of money"
2. "Amount of money at his command"
3. "Custom"
4. Price of "a rival commodity" (to avoid "cheapening of the supply of a rival commodity")

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form; even the page number is the same in the fifth and eighth editions (p. 62).

In both editions of *The Economics of Industry*, subsection 4 in Book II, "Normal Value," chap. i, "Definitions. Law of Demand," contains essentially the same material as the subsection on "value" in the *Principles* referred to in the preceding paragraph, including the following statement: "But while examining the theory of Normal value we shall, for convenience, assume that the purchasing power of money remains unchanged. So that a rise or fall in the price of a thing will always mean a rise or fall in its general purchasing power or exchange value" (pp. 68–69). No corresponding statement appears in *The Pure Theory*.

The italicized parenthesis in the second quotation is identical in the second and all later editions of the *Principles*. The remainder of the quotation was worded as follows in the second edition: "An increase in the amount of a thing that a person has will, other things being equal . . . diminish his Marginal Demand-price for it" (p. 152). In the third edition, the words "marginal" and "demand" were not capitalized, and the hyphen was eliminated after "Demand" (p. 170). In the fourth edition the end of the statement was expanded to read, "diminish the price which he will pay for a little more of it: or in other words diminishes his magrinal demand price for it" (pp. 169–70). In the fifth edition the quotation takes its present form, except for the addition of a comma, even the page number being the same as in the eighth edition (p. 95). In all editions from the second on, the indicated quotations are in Book III, chap. iii, the chapter first introducing the demand curve. This chapter is entitled "The Law of Demand" in the second and third editions, "Gradations of Demand" in the fourth, and "Gradations of Consumers' Demand" in the fifth and later editions.

The absence of the statement from the first edition reflects a difference in exposition, not in substance. As noted above, an explicit statement that the purchasing power of money is assumed constant appears in the chapter on "The Law of Demand" in the first edition. In all editions this chapter contains a statement covering the second part of the italicized parenthesis, which is worded as follows in the first edition: "Every increase in his resources increases the price which he is willing to pay for any given pleasure. And in the same way every diminution of his resources increases the marginal utility of money to him, and diminishes the price that he is willing to pay for any pleasure" (p. 156). The only change in this statement in later editions was the substitution of "benefit" for "pleasure" (8th ed., p. 96).

The Economics of Industry also contains a statement anticipating the second part of the italicized parenthesis: "The price which he is willing to pay for a thing depends not only on its utility to him but also on *his means;* that is, the amount of money or general purchasing power at his disposal" (p. 70).

In all editions of the *Principles* the statement corresponding to the third quotation is in the final subsection of the chapter first introducing the demand curve (1st ed., Book III, chap. ii; in later editions, Book III, chap. iii). In the first edition it reads: "It must be remembered that the demand schedule gives the prices at which various quantities of a thing can be sold in a market during a given time and under given conditions. If the conditions vary in any respect the figures of the schedule will probably require to be changed. One condition which it is especially important to watch is the price of rival commodities, that is, of commodities which can be used as substitutes for it" (p. 160). A footnote is attached to the word "rival," the first sentence of which reads: "Or to use Jevons' phrase (*Theory of Political Economy*, Ch. IV), commodities that are nearly 'equivalent' " (1st ed., p. 160, n. 2).

The part of the second sentence of the third quotation following the semicolon assumed its final form in the second edition (p. 157), the footnote reference to Jevons being dropped. The rest of the quotation is the same in the second and third editions as in the first and assumes its final form in the fourth (p. 174). The change made in the second sentence from the first to the second edition argues that the list was not intended to be exhaustive, but illustrative. No change in substance is involved (see 1st ed., p. 155). In all editions the quoted statement is followed by the example of tea and coffee to illustrate the necessity of assuming the prices of rival commodities to be known; in the second edition the example of gas and electricity was added, and in the third edition the example of different varieties of tea. The passage itself, the changes in it, and the examples all indicate that Marshall considered the price of "rival" commodities particularly important. The examples, together with the footnote in the first edition, make it clear that he meant "close" rivals.

For a statement in the *Pure Theory* covering the substance of these quotations, except the constancy of the purchasing power of money, see n. 38, below.

5. Range of rival commodities available (to avoid "invention of a new one")[38]

1. *The current interpretation.*—The current interpretation of Marshall's demand curve treats item 2 as referring to the money income of the group of purchasers to whom the demand curve relates, item 3 to their tastes and preferences, and item 4 to the price of every other commodity rather than of *rival* commodities alone. It ignores entirely items 1 and 5.

Item 2 is not entirely unambiguous. It might be interpreted as referring to the cash balances of the purchasers or to their wealth instead of, or in addition to, their income. On the whole, the most reasonable course seems to be to interpret it as referring to both income and wealth,[39] particularly since wealth qualifies for the list of "other things" by virtue of its possible importance as a factor affecting consumption. This expansion of the current interpretation does not alter it materially; it merely transfers "wealth" from the category of "other things" implicitly supposed to be the same to the list of things mentioned explicitly.

[38] The adequacy of this list as a summary of Marshall's views may be checked by comparing it with two others in Marshall's writings. In *The Pure Theory of (Domestic) Values*, he writes: "The periods with which we are concerned . . . are sufficiently long to eliminate . . . casual disturbances. . . . But they are sufficiently short to exclude fundamental changes in the circumstances of demand and in those of supply. On the side of demand for the ware in question it is requisite that the periods should not include (i) any very great change in the prosperity and purchasing power of the community; (ii) any important changes in the fashions which affect the use of the ware; (iii) the invention or the great cheapening of any other ware which comes to be used largely as a substitute for it; (iv) the deficiency of the supply of any ware for which the ware in question may be used as a substitute, whether this deficiency be occasioned by bad harvests, by war, or by the imposition of customs or excise taxes; (v) a sudden large requirement for the commodity, as e.g. for ropes in the breaking out of a maritime war; (vi) the discovery of new means of utilizing the ware, or the opening up of important markets in which it can be sold" (p. 15).

Item i in this list presumably corresponds with 2 in my list; ii corresponds with 3, and iii and iv with 4 and 5, iii excluding a fall in the price of a rival commodity and iv a rise. Items v and the first part of vi would seem to be contained in 3 and largely redundant with ii. The rest of vi is presumably covered by the restriction of the discussion to a demand curve for a particular market.

The other list is in Marshall's discussion in the *Principles* of the difficulties of the statistical study of demand (Book III, chap. iv), where he writes: "Other things seldom are equal in fact over periods of time sufficiently long for the collection of full and trustworthy statistics. . . . To begin with, [*a*] the purchasing power of money is continually changing. . . . Next come the changes in [*b*] the general prosperity and in the total purchasing power at the disposal of the community at large. . . . Next come the changes due to [*c*] the gradual growth of population and wealth. . . . Next, allowance must be made for changes in [*d*] fashion, and taste and habit, for [*e*] the opening out of new uses of a commodity, for [*f*] the discovery or improvement or cheapening of other things that can be applied to the same uses with it" (*Principles*, pp. 109–10; letters in brackets added). This statement dates from the first edition (pp. 170–71); only trivial editorial changes were made in later editions.

Item *a* in this list corresponds with 1 in my list; *b* with 2; *d* and presumably *e* with 3; and *f* with 4 and 5. Item *c* is presumably in part covered by restriction of the discussion to a demand curve for a particular market; in part it contains an item that may deserve to be added to the list, namely, "wealth." The wording of *f* is ambiguous, since it could refer to substitutes for the good in question, to complements, or to both. The subsequent text and the examples cited make it clear that it refers to substitutes; one example, of petroleum and petroleum lamps, itself ambiguously worded, suggests that it may refer to complements as well.

[39] In the quotations from Book III, chap. iv, in the preceding footnote, "wealth" is mentioned explicitly, though separately from "general prosperity" and "total purchasing power." See also the quotations in the fourth and fifth paragraphs of n. 37. Marshall repeatedly refers to "rich" and "poor" rather than to high- and low-income people (e.g., pp. 19, 95, 98). However, in an illustrative case, a rich man and a poor man are identified by their annual incomes (p. 19). And in Book III, chap. vi, he remarks: "We have throughout this and preceding chapters spoken of the rich, the middle classes and the poor as having respectively large, medium and small incomes—not possessions" (p. 134).

Item 3 requires no discussion, since the only reasonable interpretation of it is that it refers to tastes and preferences.[40]

The important defect of the current interpretation is its treatment of item 4, which is, in turn, responsible for the neglect of items 1 and 5. "Rival commodity" is replaced by, or read to mean, "any other commodity," and hence item 4 is taken to mean that the price of every other commodity is to be supposed the same. For example, Henry Schultz says, as if it were obvious and without citing any statements of Marshall: "Marshall also assumes, in giving definite form to the law of demand for any one commodity, that the prices of all other commodities remain constant."[41] Numerous other statements to the same effect could be cited. It is an amusing commentary on our capacity for self-delusion that the only references to Marshall for support that I have seen are to the page containing the third quotation in Section IV*a* above—the source of the words quoted in item 4.[42] The first set of italicized words in that quotation are the only words on the page even remotely supporting the substitution of "any other" for rival. The specific examples that follow the quotation—tea and coffee, gas and electric lighting, different varieties of tea, beef and mutton—make it clear that Marshall was using the word "rival" in a narrow sense and not in that broad sense in which it may be said that all commodities are "rivals" for the consumer's income.[43] Whatever the merits of the current interpretation, it cannot be found explicitly in Marshall.

The interpretation of item 4 as referring to all other commodities makes item 5 unnecessary and contradicts item 1. Item 5 is unnecessary because the introduction of a new commodity is equivalent to a decline in its price from infinity to a finite amount; hence is ruled out if the price of every other commodity is to be unchanged. Item 1 is contradicted because, if all other prices are unchanged, the purchasing power of money will be lower, the higher the price of the commodity in question. The purchasing power of money cannot, therefore, be the same for all points on the demand curve.

The redundancy of item 5 on this interpretation of item 4 is unimportant; this item is in a list that is illustrative rather than exhaustive, and there is no reason why Marshall should have scru-

[40] See n. 38, above. In discussing the law of diminishing marginal utility, Marshall says: "We do not suppose time to be allowed for any alteration in the character or tastes of the man himself" (p. 94).

[41] *Op. cit.*, p. 53. Immediately after making this statement he quotes from Edgeworth's article on "Demand Curves" cited in n. 5, above, not as evidence for the validity of his interpretation of Marshall but rather as an indication of the difficulties that it raises.

[42] Joan Robinson states without citation: "Marshall instructs us to draw up a demand schedule on the assumption that the prices of all other things are fixed" (*The Economics of Imperfect Competition* [London: Macmillan & Co., Ltd., 1934], p. 20). Paul Samuelson says, also without citation: "All other prices and income are held constant by *ceteris paribus* assumptions" in the "Marshallian partial equilibrium demand functions" (*Foundations of Economic Analysis* [Cambridge: Harvard University Press, 1947], p. 97). In an unpublished exposition of income and substitution effects prepared for class use about 1939, I stated, also without citation: "There is no question but that it [the Marshallian demand curve] was not intended to be . . . interpreted" as "showing the effect of *compensated* variations in price." Similar statements, all citing p. 100 of the *Principles* as authority, are made by Robert Triffin, *Monopolistic Competition and General Equilibrium Theory* (Cambridge: Harvard University Press, 1940), p. 44; Ruby Turner Norris, *The Theory of Consumer's Demand* (New Haven: Yale University Press, 1941), p. 82; and Weintraub, *op. cit.*, p. 539.

[43] If any doubt remains, it is removed by the footnote in the first edition attached to the word "rival" referring to Jevons' phrase "commodities that are nearly 'equivalent'" (see n. 37, above).

pulously avoided overlapping. The logical inconsistency between items 1 and 4 cannot, however, be dismissed so lightly. Retention of the current interpretation requires either that item 1 be eliminated, on the grounds that the quotations on which it is based are exceptional and peripheral, or that Marshall be convicted of logical inconsistency on a fundamental point in his theory of demand.[44] Item 1 cannot, I think, be eliminated. The constancy of the purchasing power of money is clearly fundamental in Marshall's thought, probably more fundamental than any other item on our list.[45]

One excuse for retaining the current interpretation of Marshall, despite the logical inconsistency that it introduces, is to suppose that Marshall intended to restrict the use of his demand curve to commodities that account for only a small fraction of total expenditures. A change in the price of such a commodity would have only a small effect on the purchasing power of money, and it could be argued that Marshall neglected it as a "second-order effect." On this rationalization, item 1 becomes redundant, but, in the limit, not logically inconsistent with an item 4 taken to refer to all other commodities.

I do not believe that Marshall intended to restrict the use of the demand curve to commodities accounting for only a small fraction of total expenditure. He speaks of a demand curve for wheat (p. 106), for houseroom (p. 107), and for other commodities that he cannot have regarded as unimportant. He first explicitly introduces the restriction to unimportant commodities in connection with his discussion of consumer's surplus, which comes well after the initial discussion of the demand curve—in the eighth edition, three chapters later; and the restriction is repeated at most points at which the argument depends on it. At one point the restriction is said to be "generally," not universally, justifiable. This evidence may not be conclusive but it certainly establishes a strong presumption that Marshall did not intend the restriction to carry over to all uses of the demand curve.[46]

[44] The extent to which the current interpretation dominates economic thought could not be more strikingly illustrated than by the fact that so acute an economic theorist as J. R. Hicks can write: "No doubt it [the constancy of the marginal utility of money] was . . . associated in his [Marshall's] mind with the assumption of a constant value of money (constant prices of other consumers' goods than the one, or sometimes ones, in question)" ("The Rehabilitation of Consumers' Surplus," p. 109). Hicks here treats constancy of all other prices as an alternative statement of item 1, when, in fact, it is logically inconsistent with item 1.

[45] See nn. 37 and 38, above. Note also that constancy of the purchasing power of money was a standard assumption of economic theory long before Marshall's day. It was made by Ricardo in his price theory, and Marshall refers to Cournot's discussion of the reasons for making this assumption (see Marshall, *Principles*, pp. ix, 62; Augustin Cournot, *Researches into the Mathematical Principles of the Theory of Wealth* [*1838*], Nathaniel Bacon translation [New York: Macmillan Co., 1897], p. 26).

[46] In connection with the discussion of consumer's surplus and the assumption of a constant marginal utility of money implicit in that discussion, Marshall says: "The assumption . . . underlies our whole reasoning, that the expenditure on any one thing . . . is only a small part of his whole expenditure" (p. 842). The first sentence of the paragraph from which this quotation is taken explicitly limits it to "the discussion of consumers' surplus" (p. 842). The quotation is followed by a cross-reference to the part of Marshall's famous analysis of the process by which equilibrium is reached in a corn-market in which he discusses "the latent assumption, that the dealers' willingness to spend money is nearly constant throughout" (p. 334). "This assumption," he says, "is justifiable with regard to most of the market dealings with which we are practically concerned. When a person buys anything for his own consumption, he generally spends on it a small part of his total resources" (p. 335).

Nowhere in Book III, chap. iii, does Marshall explicitly restrict his discussion to unimportant commodities. The one statement in that chapter that might be regarded as so restricting the dis-

It should be noted that Marshall's explicit introduction of the restriction to unimportant commodities has no bearing on the relative validity of the two interpretations of his demand curve. The restriction is necessary on either of the two interpretations at each point at which Marshall explicitly makes it. So the restriction cannot be regarded as called for by the inconsistency of items 1 and 4 on the current interpretation of 4.

2. *The alternative interpretation.*—My interpretation of the Marshallian demand curve resolves almost all the difficulties that plague the current interpretation, since it accepts at face value the five "other things" listed at the beginning of Section IV*b*. Marshall's words can be taken to mean what they say without uncomfortable stretching, and there is no logical inconsistency in the constancy of both item 1, the purchasing power of money, and item 4, the prices of rival commodities. Item 5, the range of rival commodities available, is still redundant, since, if "rival" has the same meaning in 4 and 5, the invention of a new rival commodity means a change in its price from infinity to a finite value.

My interpretation explains also the precise wording of the second quotation in Section IV*a*, which reads, in part: "The larger the amount of a thing that a person has the less . . . will be the price which he will pay for a little more of it." This is a curious form of phrasing on the current interpretation. Why emphasize the amount of a thing that a person has and the marginal expenditure that he can be induced to make rather than the amount he purchases and the average price he pays? On my interpretation, this phrasing follows directly from the argument of Section I*c* above (and Note II of Marshall's Mathematical Appendix), according to which a demand curve constructed on my interpretation can be viewed as showing the maximum price per unit that a person can be induced to pay for successive increments of the commodity.

One minor puzzle remains on my interpretation. Why does Marshall restrict his attention to "rival" commodities? Why not to "closely related" commodities, whether rivals or complements? His use of the word "rivals" in discussing the demand curve is apparently not a mere verbal accident. He uses the word repeatedly; almost all his examples deal with the effect of, or through, substitutes. I have no very good answer to this puzzle; the only one that seems at all persuasive is that he thought the concept of "joint demand" and the associated analytical apparatus better suited to problems involving complementary goods.[47]

My interpretation follows so directly

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cussion is the statement on p. 95 that "the marginal utility of money to him is a fixed quantity." But the context argues and Note II in the Mathematical Appendix demonstrates that this is merely a verbal statement of an identity (if income is unchanged, so is marginal utility of money), and thus is not really relevant to the issue. In the eighth edition, Note II is referred to only at the end of the subsection following the paragraph containing the passage quoted. However, in the first edition, the corresponding note (Note III) is referred to at the end of the paragraph containing the passage quoted, and hence clearly covers it (pp. 155–56, 737–38).

The above quotations are essentially unchanged from the first edition on. The restriction to unimportant commodities is, however, mentioned neither in Marshall and Marshall, *Economics of Industry*, nor in the *Pure Theory*.

[47] In Note VII of the Mathematical Appendix, Marshall qualifies a suggested formula for combining consumer's surplus from different commodities by saying: "if we could find a plan for grouping together in one common demand curve all those things which satisfy the same wants, and are rivals; and also for every group of things of which the services are complementary (see Book V, chap. vi) . . ." (p. 842). Book V, chap. vi, contains the discussion of joint demand. The qualification quoted appears first in the third edition.

from Marshall's words that further defense of it would be unnecessary were it not for the unquestioned dominance of the current interpretation in the economic thinking and writing of the past half-century. This circumstance explains the presentation of additional textual evidence bearing on the validity of the alternative interpretation.

c) COUNTEREVIDENCE FROM THE TEXT OF THE "PRINCIPLES"

I have been able to find only one passage in the text of the eighth edition of the *Principles* that is in any way inconsistent with my interpretation of Marshall. This is the celebrated passage, adverted to above, which deals with the so-called "Giffen phenomenon" and which was first introduced in the third edition:

> For instance, as Sir R. Giffen has pointed out, a rise in the price of bread makes so large a drain on the resources of the *poorer labouring families* and raises so much the marginal utility of money to them, that they are forced to curtail their consumption of meat and the more expensive farinaceous foods: and bread being still the cheapest food which they can get and will take, they consume more, and not less of it [p. 132; italics added].

This passage clearly offsets an income effect against a substitution effect, whereas, on my interpretation of Marshall, real income is the same at all points on the demand curve, so there is no "income effect" (see Sec. II*b*, above). The passage is thus in the spirit of the current interpretation. Yet the words I have italicized indicate that it does not necessarily contradict my interpretation of Marshall. The purchasing power of money and the real income of the community at large may remain constant; yet the real income of a particular group in the community that has a special consumption pattern may be adversely affected by the rise in the price of a particular commodity.[48]

d) THE EVIDENCE OF THE MATHEMATICAL APPENDIX

The Mathematical Appendix to the *Principles* confirms and extends the evidence already presented from the text of the *Principles* and from Marshall's other writings. Note II (III in the first edition) explicitly derives a relation between price and quantity demanded that is identical with a demand curve on my interpretation, in which real income is kept constant by compensating variations in money income. Indeed, my derivation of such a demand curve in Section I*c* above is a verbal paraphrase of Marshall's mathematics. Marshall does not explicitly say that the relation he derives is a demand curve, but Note II is attached to his initial discussion of the demand curve (Book III, chap. iii, in the eighth edition) and is given as the authority for statements made about the demand curve; hence there can be no doubt that it presents the pure theory of his demand curve.

In all editions of the *Principles* Note VI, attached to Marshall's discussion of consumer's surplus, contains a sentence that is definitely wrong on the current interpretation of his demand curve but correct on my interpretation.

Finally, a sentence added to Note VI in the third edition, referred to in the text of the *Principles* in connection with the material added on the Giffen phenomenon, contains an implicit mathematical proposition that is correct on the current interpretation but incorrect on my interpretation. The mathematical point in question is considerably more

[48] See Marshall's explicit discussion of, and emphasis on, this possibility in "Remedies for Fluctuations of General Prices" (1887), *Memorials*, p. 207.

subtle than those referred to in the two preceding paragraphs, so it cannot be given the same weight.

These two notes are examined in some detail in the appendix to this paper, to which the reader is referred for proof of the above statements.

c) A SYNTHESIS OF THE EVIDENCE

There are two differences between the current interpretation of Marshall's demand curve and my interpretation: (1) On the current interpretation, account is taken of the price of each other commodity individually; on my interpretation, only of the average price of all commodities other than the one in question and its close rivals. (2) On the current interpretation, real income varies along the demand curve with the price of the good in question; on my interpretation, real income is constant along the demand curve.

On the first, and less important, point, it is mathematically convenient to consider each other price separately, and this procedure might well have recommended itself to the writer of mathematical Notes XIV and XXI. On the other hand, it is impossible to consider each price separately in a practical analysis; so the use of an average price would clearly have recommended itself to the writer of the text of the *Principles* and is entirely in the spirit of Marshall's explicit methodological statements (see Sec. II*a*, above). Marshall does not discuss this point explicitly; hence the textual evidence is all indirect.

On the second and basic point of difference the evidence leaves little room for doubt: Marshall's theory of demand, in the form in which it is presented in the first edition of the *Principles*, is explicitly based on constancy of real income along the demand curve. This interpretation not only is consistent with both the letter and the spirit of the entire text of the first edition of the *Principles* but is almost conclusively established by the evidence cited above from two notes in the Mathematical Appendix of the first edition. In his determined effort to be persuasive and to make his work accessible to educated laymen, Marshall might well have been vague in his verbal presentation, though even there it seems unlikely that he would have been logically inconsistent. It is hardly credible that he would have been not merely vague but downright wrong on simple mathematical points stated in mathematical language, especially since the mathematical points in question could hardly even have arisen if he had been explicitly using the current interpretation of the demand curve.

I am inclined to believe, however, that by the time Marshall made the revisions incorporated in the third edition of the *Principles*—presumably between 1891, when the second edition appeared, and 1895, when the third edition appeared—he had himself been influenced by the current interpretation, probably without realizing that it was different from his own. This conjecture is based primarily on the two passages cited above as inconsistent with my interpretation: the passage dealing with the Giffen phenomenon and the last sentence of Note VI of the Mathematical Appendix. Both were added in the third edition—and these are the only passages I have been able to find in any edition of the *Principles* that fit the current interpretation better than they fit my interpretation. Further, both show some evidence of confusion about the fine points of his theory of demand (see last paragraph of appendix to this paper).

The hypothesis that Marshall did not

recognize the contradiction between the current interpretation and his earlier work would hardly be tenable if the lapse of time between the work incorporated in the first and the third editions of the *Principles* were as short as between their publication. But, as already noted, this is not the case. The essence of both his theory of demand and his analysis of consumer's surplus is contained in the *Pure Theory of Domestic Values*, which, though not printed until 1879, "must have been substantially complete about 1873."[49] The one important point in the theory of demand that is not in the *Pure Theory*—explicit mention of constancy of the purchasing power of money—is in the 1879 *Economics of Industry*. The only important addition in the *Principles* is the concept of "elasticity of demand"; and even this concept, which is not relevant to the present problem, was worked out in 1881–82.[50] No important substantive changes were made in the theory of demand in successive editions of the *Principles*, though the exposition was amplified and rearranged, the wording changed in detail, and some examples modified. The only important change of substance introduced into the discussion of consumer's surplus (in the third edition) was in connection with a point that has no bearing on the present issue.[51]

Marshall himself writes: "My main position as to the theory of value and distribution was practically completed in the years 1867 to 1870. . . . By this time [from the context, 1874] I had practically completed the whole of the substance of my Mathematical Appendix."[52] Thus Marshall appears to have completed his fundamental work on the theory of demand in the early 1870's and to have made no important substantive changes thereafter. The third edition appeared some twenty or more years later—an ample lapse of time for the precise details of an essentially mathematical analysis to have become vague and their difference from a superficially similar set of details to pass unnoticed. This seems especially plausible in view of the acceptance of the current interpretation by others and the absence of controversy about it.

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[49] Keynes, *Memorials*, p. 23.

[50] *Ibid.*, p. 39, n. 3.

[51] This change does not reflect favorably on Marshall's willingness to admit error. The first edition states: "Subject to these corrections then we may regard the aggregate of the money measures of the total utility of wealth as a fair measure of that part of the happiness which is dependent on wealth" (pp. 179–80), the corrections referred to being for "differences in the wealth of different purchasers" (p. 178) and "elements of collective wealth which are apt to be overlooked" (p. 179). A footnote to the first quotation refers to mathematical Note VII, in which he says, subject to the same two qualifications: "if $a_1, a_2, a_3 \ldots$ be the amounts consumed of the several commodities of which $b_1, b_2, b_3 \ldots$ are necessary for existence, if $y = f_1(x)$, $y = f_2(x)$, $y = f_3(x) \ldots$ be the equations to their demand curves . . . , then the total utility of his wealth, subsistence being taken for granted, is represented by

$$\sum \int_b^a f(x)dx"$$

(1st ed., p. 741).

The eighth edition does not contain the first statement. Instead, the text contains an explicit warning against adding consumer's surpluses for different commodities, and a footnote says: "Some ambiguous phrases in earlier editions appear to have suggested to some readers the opposite opinion" (p. 131). Note VII in the Mathematical Appendix was modified by replacing "his wealth" by "income" and, of more importance, "is represented" by "might be represented" and by adding after the formula the significant qualification, "if we could find a plan for grouping together in one common demand curve all those things which satisfy the same wants, and are rivals; and also for every group of things of which the services are complementary. . . . But we cannot do this; and therefore the formula remains a mere general expression, having no practical application" (p. 842). As noted, these changes date from the third edition.

[52] Letter to J. B. Clark, *Memorials*, p. 416.

Further circumstantial evidence that Marshall did not recognize the contradiction between the current interpretation and his earlier work is provided by the apparent absence of any explicit discussion of the question in the writings of either Marshall or the more prominent of his students or even of any comments that could reasonably be interpreted as implying recognition of the existence of alternative interpretations of the demand curve. Yet, as noted earlier (n. 5), the current interpretation is explicitly given by Edgeworth as early as 1894 in an article on "Demand Curves" in *Palgrave's Dictionary of Political Economy* that Marshall must be presumed to have read. Though the assumption of constant prices of commodities other than the one in question is not explicitly attributed to Marshall, most of the article is based on Marshall; and there is no suggestion that this assumption does not apply to Marshall's demand curve. Further, Walras' definition of the demand curve, which presumably influenced Edgeworth, is identical with the current interpretation of Marshall's demand curve, and Marshall refers to Walras several times in the first edition of the *Principles*, though it seems clear that Marshall developed his theory of demand independently of Walras.[53] So Marshall must have been exposed to a definition of the demand curve corresponding to the current interpretation at a time when he was still making substantial revisions in the *Principles*. If he had recognized that this interpretation was incorrect, would he not have taken the opportunity to clarify his statements in later editions?

[53] *Principles* (1st ed.), pp. xi, xii, 425; Keynes, *Memorials*, pp. 19–24; Marshall's letter to J. B. Clark, *ibid.*, pp. 416–18.

V. ALTERNATIVE CONCEPTIONS OF ECONOMIC THEORY

There remains the mystery how the current interpretation of Marshall's demand curve gained such unquestioned dominance at so early a date and retained it so long, not only as an interpretation of Marshall, but also as "the" definition of "the" demand curve.

One obvious explanation is that mathematical economists were more likely than others to state explicitly and precisely their assumptions about the behavior of other prices; that mathematical economists were likely to be familiar with Walras' independent definition and to take it as a point of departure; and that, in any event, the current interpretation is mathematically more convenient. Other economists, it could be argued, followed the lead of the mathematical economists, and thus the current interpretation was taken for granted and accepted without question.

This explanation seems to me a significant part of the answer; however, I do not believe that it is the entire answer. If, as I have argued above, my interpretation of Marshall is more useful for most practical problems, why has its use been so rarely proposed; why has there been no general feeling of dissatisfaction with the current interpretation? There must, it would seem, be something about the role that has been assigned to economic theory that has made the current interpretation acceptable.

I am inclined to believe that this is, in fact, the case; that, by slow and gradual steps, the role assigned to economic theory has altered in the course of time until today we assign a substantially different role to theory than Marshall did. We curtsy to Marshall, but we walk with Walras.

The distinction commonly drawn between Marshall and Walras is that Marshall dealt with "partial equilibrium," Walras with "general equilibrium." This distinction is, I believe, false and unimportant. Marshall and Walras alike dealt with general equilibrium; partial equilibrium analysis as usually conceived is but a special kind of general equilibrium analysis—unless, indeed, partial equilibrium analysis is taken to mean erroneous general equilibrium analysis. Marshall wrote to J. B. Clark in 1908: "My whole life has been and will be given to presenting in realistic form as much as I can of my Note XXI."[54] Note XXI, essentially unchanged from the first edition of the *Principles* to the last, presents a system of equations of general equilibrium. It ends with the sentence: "Thus, however complex the problem may become, we can see that it is theoretically determinate, because the number of unknowns is always exactly equal to the number of equations which we obtain."[55] The explanation given above why Marshall might have decided to hold the purchasing power of money constant was entirely in terms of constructing the demand curve so that it would be consistent with general equilibrium in those parts of the system not under direct study.

The important distinction between the conceptions of economic theory implicit in Marshall and Walras lies in the purpose for which the theory is constructed and used. To Marshall—to repeat an expression quoted earlier—economic theory is "an engine for the discovery of concrete truth." The "economic organon" introduces "systematic and organized methods of reasoning." Marshall wrote:

> Facts by themselves are silent. . . . The most reckless and treacherous of all theorists is he who professes to let facts and figures speak for themselves, who keeps in the background the part he has played, perhaps unconsciously, in selecting and grouping them, and in suggesting the argument *post hoc ergo propter hoc*. . . . The economist . . . must be suspicious of any direct light that the past is said to throw on problems of the present. He must stand fast by the more laborious plan of interrogating facts in order to learn the manner of action of causes singly and in combination, applying this knowledge to build up the organon of economic theory, and then making use of the aid of the organon in dealing with the economic side of social problems.[56]

Economic theory, in this view, has two intermingled roles: to provide "systematic and organized methods of reasoning" about economic problems; to provide a body of substantive hypotheses, based on factual evidence, about the "manner of action of causes." In both roles the test of the theory is its value in explaining facts, in predicting the consequences of changes in the economic environment. Abstractness, generality, mathematical elegance—these are all secondary, themselves to be judged by the test of application. The counting of equations and unknowns is a check on the completeness of reasoning, the beginning of analysis, not an end in itself.

Doubtless, most modern economic theorists would accept these general statements of the objectives of economic theory. But our work belies our professions. Abstractness, generality, and mathematical elegance have in some measure become ends in themselves, criteria by which to judge economic theory. Facts are to be described, not explained. Theory is to be tested by the accuracy of its "assumptions" as photographic descrip-

[54] *Memorials*, p. 417.

[55] *Principles*, p. 856. This note was numbered XX in the first edition.

[56] The quotations are all taken from Marshall, "The Present Position of Economics" (1885), *Memorials*, pp. 159, 161, 164, 166, 168, 171.

tions of reality, not by the correctness of the predictions that can be derived from it. From this viewpoint the current interpretation of the demand curve is clearly the better. It is more general and elegant to include the price of every commodity in the universe in the demand function rather than the average price of a residual group. Any price may affect any other, so a demand equation including every price is a more accurate photographic description. Of course, it cannot be used in discovering "concrete truth"; it contains no empirical generalization that is capable of being contradicted—but these are Marshallian objections. From the "Walrasian" viewpoint, to take one other example from recent developments in economic theory, it is a gain to eliminate the concept of an "industry," to take the individual firm as the unit of analysis, to treat each firm as a monopoly, to confine all analysis to either the economics of the individual firm or to a general equilibrium analysis of the economy as a whole.[57] From the Marshallian viewpoint this logical terminus of monopolistic competition analysis is a blind alley. Its categories are rigid, determined not by the problem at hand but by mathematical considerations. It yields no predictions, summarizes no empirical generalizations, provides no useful framework of analysis.

Of course, it would be an overstatement to characterize all modern economic theory as "Walrasian" in this sense. For example, Keynes's theory of employment, whatever its merits or demerits on other grounds, is Marshallian in method. It is a general equilibrium theory containing important empirical content and constructed to facilitate meaningful prediction. On the other hand, much recent work based on Keynes's theory of employment is Walrasian.[58]

[57] See Triffin, *op. cit.*, pp. 188–89.

VI. CONCLUSION

Modern economic theory typically defines the demand curve as showing the relation between the quantity of a commodity demanded and its price for given tastes, money income, and prices of other commodities. This definition has also been uniformly accepted as a correct interpretation of the demand curve defined and used by Alfred Marshall in his *Principles of Economics*. Rarely has the view been expressed that a different definition would be preferable.

Despite its unquestioned acceptance for over half a century, this interpretation of Marshall is, in my view, wrong. Marshall's early writings, the text of the *Principles*, and, even more definitely, the Mathematical Appendix provide almost conclusive proof that Marshall's demand curve differs in two respects from the one commonly used and attributed to him: first, commodities other than the one in question and its close rivals are treated as a group rather than individually, and only their average price is explicitly taken into account; second, and far more important, real income is considered the same at all points on the demand curve, whereas constant money income and other prices imply a higher real income, the lower the price of the commodity in question. Two variants of Marshall's demand curve can be distinguished: one, employed in the text of the *Principles*, uses variations in the prices of other commodities to compensate for variations in the price of the commodity in question and thereby keeps the purchasing power of money constant; the other, employed in the Mathematical

[58] O. Lange, *Price Flexibility and Employment* (Bloomington, Ind.: Principia Press, 1944), is perhaps as good an example as any.

Appendix, uses variations in money income to compensate for variations in the price of the commodity in question.

The only textual evidence that conflicts with this interpretation is a passage in the text and a related sentence in the Mathematical Appendix that were added to the third edition of the *Principles*. The inconsistency of these with the rest of the *Principles* can be explained by the hypothesis that Marshall himself was after a point influenced by the current interpretation of the demand curve without recognizing its inconsistency with his earlier work. Some circumstantial evidence also supports this hypothesis.

The alternative interpretation of the demand curve not only is faithful to both the letter and the spirit of Marshall's work but also is more useful for the analysis of concrete problems than is the demand curve commonly employed. The acceptance of a less useful definition seems to me to be a consequence of a changed conception of the role of theory in economic analysis. The current interpretation of the demand curve is Walrasian; and so is current economic theory in general.

APPENDIX ON TWO NOTES IN THE MATHEMATICAL APPENDIX TO THE *PRINCIPLES*

1. NOTE II OF THE EIGHTH EDITION

This note is numbered III in the first edition of the *Principles*, II in the rest. In the first edition the relevant parts are worded as follows (pp. 737–38):

"If m is the amount of money or general purchasing power at a person's disposal at any time, and μ represents its total utility to him, then $d\mu/dm$ represents the marginal utility of money to him.

"If p is the price which he is just willing to pay for an amount x of the commodity which gives him a total pleasure u, then

$$\frac{d\mu}{dm}\Delta p = \Delta u \; ; \quad \text{and} \quad \frac{d\mu}{dm}\frac{dp}{dx} = \frac{du}{dx} \ldots .$$

"Every increase in his means diminishes the marginal utility of money to him; . . . .

"Therefore, du/dx, the marginal utility to him of an amount x of a commodity remaining unchanged, an increase in his means . . . increases dp/dx, that is, the rate at which he is willing to pay for further supplies of it. Treating u as a variable, that is to say, allowing for possible variations in the person's liking for the commodity in question, we may regard dp/dx as a function of m, u, and x. . . ."

The wording in the eighth edition is identical except that "marginal utility of money" is replaced by "marginal degree of utility of money" and that "du/dx" and the words "Treating u . . . in question" are omitted from the last paragraph quoted (pp. 838–39). The changes were first made in the third edition.

In the second sentence of this note, the word "price" is to be interpreted as "total amount" not as "price per unit." This is clear from the context and is demonstrated by the equation that follows and the designation of dp/dx as "the rate at which he is willing to pay for further supplies of it." The words "just willing" in the second sentence and the equations that follow demonstrate that p is the maximum amount he can pay for an amount x and have the same utility as if he had none of the commodity. Thus Marshall is describing a process like that outlined in Section I*c* of this paper, whereby the maximum possible amount is extracted from the individual for each successive increment of the commodity, the individual retaining the same "real income," that is, remaining on the same indifference curve, throughout the process.

The last sentence quoted shows that u is to be regarded as a parameter to allow for changes in tastes. The rest of that sentence simply describes a function like that obtained by eliminating y' from equations (5) and (6) of note 6 of this paper. The parameter m in Marshall's function takes the place of U_0 in our footnote, since dp/dx is still to be regarded as the price per unit paid for an additional increment of the commodity rather than as the price per unit at which any amount can be purchased. In consequence, no explicit statement is needed as yet about the compensating variations in income that are implicit in Marshall's analysis.

The word "demand" does not appear in this note. But the note is attached to the chapter in the *Principles* in which Marshall first introduces the demand curve (Book III, chap. ii, in

the first edition; Book III, chap. iii, in later editions) and is cited as proof of statements about the demand curve; hence there can be no doubt that the "function" mentioned in the last sentence quoted is the counterpart of Marshall's demand curve.

I have been able to construct no interpretation of this note that would render it consistent with the current interpretation of Marshall's demand curve.

2. NOTE VI

This note has the same number in all editions. In the first edition the relevant parts are worded as follows (p. 740):

"If y be the price at which an amount x of a commodity can find purchasers in a given market, and $y = f(x)$ be the equation to the demand-curve, then the total utility of the commodity is measured by $\int_0^a f(x)\,dx$, where a is the amount consumed.

"If however an amount b of the commodity is necessary for existence, $f(x)$ will be infinite, or at least indefinitely great, for values of x less than b. We must therefore take life for granted, and estimate separately the total utility of that part of the supply of the commodity which is in excess of absolute necessaries: it is of course $\int_b^a f(x)\,dx \ldots .$

"It should be noted that, in the discussion of Consumers' Rent, we assume that the marginal utility of money to the individual purchaser is the same throughout. . . ."

Only trivial changes were made in these sentences in subsequent editions: a typographical error in the fifth edition, which remained uncorrected thereafter, substituted $f(z)$ for $f(x)$ in the second sentence; and "consumers' surplus" replaced "Consumers' Rent." In the third edition the following sentence was added at the end of the note:

"If, for any reason it be desirable to take account of the influence which his expenditure on tea exerts on the value of money to him, it is only necessary to multiply $f(x)$, within the integral given above by that function of $xf(x)$ (i.e. of the amount which he has already spent on tea) which represents the marginal utility to him of money when his stock of it has been diminished by that amount" (3d ed., p. 795). The only subsequent changes were the addition of a comma after "reason" and the deletion of the comma before "within" (8th ed., p. 842).

In its final form Note VI seems internally inconsistent: the second sentence is wrong on the current interpretation of Marshall's demand curve, correct on my interpretation; the final sentence, added in the third edition, seems correct on the current interpretation, wrong on my interpretation.

a) THE SECOND SENTENCE

The second sentence is wrong on the current interpretation, which holds money income and other prices constant along the demand curve, since the ordinate of the demand curve for any quantity x cannot then exceed money income divided by x, and this is not "indefinitely great" for a fixed value of x—say, x_0—whether x_0 is greater or less than b. True, $f(x)$ might approach infinity as x approaches zero, but this is not what Marshall says; he says it is "indefinitely great, for values of x less than b," i.e., for any particular value of x less than b—say, $x_0 = 0.99b$.

On the variant of my interpretation involving compensating variations in money income—the variant that the note numbered II in the eighth edition leads me to believe Marshall used in the Mathematical Appendix—this sentence is entirely valid. As x declines from a value larger than b, the compensating variation in money income required to keep the individual's real income the same becomes larger and larger, approaching infinity as x approaches b, the minimum amount necessary for existence. This permits the ordinate of the demand curve likewise to approach infinity as x approaches b. On the variant involving compensating variations in other prices—the one Marshall used in the text—the definition of the demand curve breaks down for values of x less than b: for a finite price of the commodity in question, sufficiently high so that the given money income could purchase only a quantity x less than b, there will exist no set of nonnegative prices for the remaining commodities that will keep the purchasing power of money constant in the sense of enabling the same money income to provide the same level of utility; money income and real income cannot both be held constant and at the same time all prices be kept nonnegative. This sentence can therefore be defended as valid on either variant of my interpretation.

One possible ground for dismissing this sentence as evidence against the current inter-

pretation is that the so-called "error" on that interpretation is of my own making, arising from a too subtle and too literal reading of the note. Marshall, it could be argued, was using "demand curve" to mean "utility curve" and $f(x)$ to mean "marginal utility," and therefore he did not consider whether the sentence would be valid if $f(x)$ were to be interpreted literally as the ordinate of the demand curve. A note that Marshall published in 1893 on "Consumer's Surplus" could be cited as evidence for this contention. In this note he quotes part of Note VI as follows: " 'If, however, an amount b of the commodity is necessary for existence, [the utility of the first element] a will be infinite.' "[19] The bracketed expression that Marshall substituted for $f(x)$ would support the notion that he was using "demand curve" and "utility curve" interchangeably.

I do not myself accept this argument; it seems to me to do much less than justice to Marshall. In the first place, I am inclined to give little weight to an incidental, explanatory, phrase inserted by Marshall as late as 1892 or 1893, some twenty years after the fundamental analysis incorporated in Note VI had been completed. I have noted above and shall presently cite evidence that Marshall may have been somewhat confused about the fine points of his own theory of demand by the early 1890's. In the second place, and more important, Marshall clearly distinguishes in the earlier notes in the Mathematical Appendix between a utility curve and a demand curve, repeatedly using the word "utility," and in the first sentence of Note VI says that "the total utility of the commodity is *measured by* $\int_0^a f(x)\,dx$" (1st ed., p. 740; italics added). If he had been using $f(x)$ to stand for marginal utility, the words I have italicized could have been omitted. Finally, Note VI, like most of the rest of the Mathematical Appendix, summarizes a subtle, closely reasoned, and by means obvious, mathematical argument, in which, so far as I know, few errors have ever been found. Is it credible that it would have been worded as loosely and carelessly as the argument being criticized requires; or that, if at one stage it had been, Marshall would have failed to see the simple mathematical error implicit in a literal reading of his words on the current interpretation of the demand curve? It seems to me far more credible that he meant what he said and that the correctness of what he said on my interpretation of his demand curve is strong evidence for that interpretation.

[19] "Consumer's Surplus," *Annals of the American Academy of Political and Social Science*, III (March, 1893), 618–21 (brackets in original). This note is a reply to some comments by Simon Patten. The letter a after the brackets which appears in the *Annals* note does not appear in the *Principles*, and I can explain it only as a typographical error.

b) THE FINAL SENTENCE

The explanation that follows of the final sentence added to Note VI in the third edition, though not completely satisfactory, is reasonably so, and I have been able to construct no other even remotely satisfactory explanation.

Let U be the utility function of the "individual purchaser" and U_x the marginal utility of x units of tea to him, i.e., the partial derivative of U with respect to x. Now the increase in utility attributable to having a rather than b units of tea—consumer's surplus in utility units—is given by

$$\int_b^a U_x\,dx, \qquad (1)$$

where the integral is computed for constant quantities of other commodities equal to the amounts consumed when a units of tea are consumed and other conditions are those corresponding to the demand curve $y = f(x)$.

At every point along the demand curve,

$$U_x = ny = n\,(x)\,f\,(x), \qquad (2)$$

where n is the marginal utility of money—itself, of course, a function of x along the demand curve. Integrating both sides of equation (2) gives

$$\int_b^a U_x\,dx = \int_b^a n\,(x)\,f\,(x)\,dx. \qquad (3)$$

The left-hand side of equation (3) is symbolically identical with equation (1); yet there is an important difference between them. In equation (1), U_x is computed, holding the quantities of other commodities constant as x varies; in equation (3), U_x is computed, holding constant whatever is held constant along the demand curve (money income and other prices on the current interpretation; real income on my interpretation). In general, quantities of other commodities vary along the demand curve (on either interpretation), and U_x may depend on the quantities of other commodities, so the

U_x in equation (3) may be numerically different from the U_x in equation (1) for a value of x other than a. This difficulty disappears if U_x is supposed to be independent of the quantities of other commodities—an assumption that Marshall pretty clearly makes as a general rule (e.g., see Nn. I and II of the Mathematical Appendix). On this assumption, then, the right-hand side of equation (3) measures consumer's surplus in utility units.

It is at this point that difficulties of interpretation arise; for the right-hand side of equation (3) is obtained by multiplying "$f(x)$ within the integral given above by that function of" x "which represents the marginal utility . . . of money." Why does Marshall say "that function of $xf(x)$" rather than of x alone? And is it valid to make this substitution? One can argue that to each value of x there corresponds a value of $f(x)$ and hence of $xf(x)$, so that the two forms of statement are equivalent: Marshall has simply made the transformation $z = xf(x)$ and converted $n(x)$ into $n(z)$. This argument is not, however, rigorous. In general, x will not be a single-valued function of z; hence to any given value of z there may correspond more than one value of x and hence more than one value of n. The two forms of statement are equivalent if and only if n is a single-valued function of z, i.e., if $n(x)$ is the same for all values of x for which $xf(x)$ is the same.

Given independence between the marginal utility of tea and the quantity of other commodities, this condition is always satisfied on the current interpretation of the demand curve but not on the alternative interpretation. Let x' stand for the quantity of a composite commodity representing all commodities other than tea, y' for its price, and $U_{x'}$, for its marginal utility. At each point on the demand curve,

$$\frac{U_x}{y} = \frac{U_{x'}}{y'} = n .$$

On the current interpretation of the demand curve, money income and the prices of other commodities are the same for all points on the demand curve. It follows that, for all values of x that yield the same value of $xf(x)$, the same amount will be spent on other commodities; so x' is the same (since y' is by definition); so $U_{x'}$ is (since, on the assumption of independence, $U_{x'}$ depends only on x'); and so n is. Marshall's use of $xf(x)$ instead of x is thus valid on the current interpretation of the demand curve.

On my interpretation, either money income varies along the demand curve, so as to keep real income constant, or other prices do; hence the preceding argument is no longer valid. That the two forms of statement are no longer always equivalent can be shown by a counter-example. If other prices are held constant and compensating variations of income are used to keep real income constant, $U = \sqrt{x} + \sqrt{x'}$ is a utility function that gives different values of n for different values of x yielding the same value of $xf(x)$. If money income is held constant and compensating variations of other prices are used to keep real income constant, $U = 3 + x - \frac{1}{10}x^2 + \sqrt{x'}$ is such a utility function. Hence Marshall's use of $xf(x)$ instead of x is invalid on either variant of the alternative interpretation.

This explanation leaves a number of Marshall's verbal statements wrong or ambiguous, whichever interpretation of the demand curve is accepted. (1) The parenthetical explanation of the meaning of $xf(x)$ seems wrong—why the word "already"? If one is thinking of going through the process of extracting as much as possible from the consumer for each successive unit of tea and is supposing the maximum price that he will pay for successive units to be given by the demand curve, then $\int_0^x f(x)dx$ and not $xf(x)$ is the amount he has "already spent on tea." If one is thinking of the amount spent on tea at a given price for tea, then $xf(x)$ is the amount spent when the price is $f(x)$, not the amount "already spent." The explanation offered above accepts the latter rendering of the parenthesis, i.e., supposes the word "already" omitted. (2) The last clause—"when his stock of money has been diminished by that amount"—is ambiguous. To make it consistent with the explanation offered above, one must add "and tea is unavailable, so that the balance is spent solely on other commodities at the prices assumed in drawing the demand curve for tea." The reference to "stock of money" suggests that Marshall was supposing money income constant and so, independently of the rest of the quotation, would tend to rule out compensating variations in money income. It should be noted that there are no such ambiguities in the original version of Note VI, either in the parts quoted above or in the parts not quoted.

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THE PURE THEORY OF GOVERNMENT FINANCE

A SUGGESTED APPROACH

JAMES M. BUCHANAN

I

A FRAMEWORK for the pure theory of government finance may be erected on either of two political foundations, which represent, in turn, two separate and opposing theories of the state. Since neither construction is entirely appropriate when applied to all the problems faced in the fiscal area, the proper methodological procedure seems to be the setting-up of alternate theories.

In the first, or what may be called the "organismic," theory, the state, including all individuals within it, is conceived as a single organic entity. In the second, the state is represented as the sum of its individual members acting in a collective capacity. The individual and the state are fundamentally opposing forces in the latter concept, while in the organic view the state, or general interest, subsumes all individual interests. The theory of government finance based upon the second concept of the state may be called the "individualistic" one.

These two approaches have not been clearly separated or distinguished in the literature of government finance. Some variant of the organismic theory normally has been applied to the public expenditure side, while the individualistic theory has been predominantly employed in considering the distribution of the tax load. Fiscal marginalism has been extended to define the optimum allocation of public expenditures among alternative uses, that is, in the pure theory of budgeting. The allocation of total tax burden among alternative sources, has, on the other hand, traditionally been discussed in terms of the relative tax pressures imposed upon individuals.[1] Such asymmetry, while perhaps appropriate in practical application, does not appear formally complete. It seems desirable to develop the two theories independently at the outset.

II

In the organismic theory the state is considered as a single decision-making unit acting for society as a whole. Presumably, it seeks to maximize some conceptually quantifiable magnitude. A major difficulty is apparent in the determination of what is to be maximized. What is the common denominator to which the alternative goals of the collective entity may be reduced for comparative purposes, analogous to the equally vague, but less elusive, "satisfaction" or "utility" for the individual? A common denominator is necessary as a starting point in order that any one "configuration of

[1] There have been, of course, certain elements of each approach present in most competent works. The individual benefits from public expenditures have never been entirely overlooked, nor have the social effects of taxation been completely neglected. The work of Hugh Dalton is representative of a complete organismic approach, with certain qualifications (*Principles of Public Finance* [9th rev. ed.; London: George Routledge & Sons, 1936]). The theory presented by the Italian school best represents the individualistic approach (cf. Antonio de Viti de Marco, *First Principles of Public Finance*, trans. E. P. Marget [London: Jonathan Cape, 1936]).

the economic system" may be regarded as better or worse than any other, or indifferent.[2] This general end of society may be called "general welfare" or "social utility"; the name is not important.

"Social utility" is a function of many variables. The maximizing process will include the manipulation of many factors which lie outside the scope of fiscal theory. The structure of the whole social organization is itself a property subject to change. Many of the variables are noneconomic; and, even among those which can be classified as economic in nature, a relatively small proportion can be embraced in fiscal theory. The nonfiscal variables, therefore, must be accepted as parameters for the smaller system, and "social utility" must be maximized, subject to the constraints imposed. The variables to be determined in the fiscal process fall into two groups—the expenditure variables and the tax variables. The amounts of public expenditure allocated to each use comprise the expenditure variables. The amounts of tax load imposed upon each economic entity or tax source (individuals, business units, estates, etc.) comprise the tax variables. There are as many variables in the fiscal system as there are expenditure outlets and tax sources. Any allocation of the total tax load represents a solution for the whole set of tax variables. Any distribution of public expenditure among competing uses indicates a fixing of values for all the expenditure variables. Included in the setting of these values is the determination of total tax load and total public expenditures.

It is the function of the "fiscal brain" to select the values of these many variables which will maximize social utility. The maximizing process consists of a simultaneous determination of all the variables on both sides. The necessary condition for a maximum is produced when the partial derivatives vanish or when a dollar's tax load upon each economic entity deducts from social utility an amount equivalent to that added by a dollar's expenditure in each line.

It is important to note that the optimum values for the tax variables cannot be determined independently except for given values for the expenditure variables. The allocation of the tax load which will maximize social utility or, in this case, minimize deductions from utility will be different for each separate distribution of total expenditure. Similarly, the distribution of expenditure which will maximize welfare will be a function of the tax variables. The relative additions to social utility provided by the offering of public services to particular groups will be dependent in part upon the relative tax loads imposed.

The principle of taxation which is appropriate in this theoretical framework is that of the Edgeworth-Pigou variety. The relevant criteria of comparison are reductions from social welfare or utility; the "least-aggregate-sacrifice" approach is the correct one. The principle of equimarginal "sacrifice" or, better, "subtraction," if values are given for the expenditure variables, provides an acceptable rule for the apportionment of the total tax burden. For each allocation of public expenditure the satisfaction of this principle will define an apportionment which will minimize the subtraction from social utility. It should be emphasized, however, that subtraction can be used only with reference to a social, not an individual, concept of utility.[3] For each

[2] P. A. Samuelson, *Foundations of Economic Analysis* (Cambridge: Harvard University Press, 1947), p. 221.

[3] In this sense the principle of equimarginal sacrifice implies nothing about the ultimate equali-

given distribution of public expenditures the necessary condition for the optimum allocation of the tax load is reached when a dollar's tax upon each economic entity deducts an equivalent amount from aggregate social utility. The economic entities for such tax comparisons may or may not be individuals in this analysis.

Symmetrically, given values for the tax variables, an allocation of total public expenditure among alternative uses can be found which will maximize social welfare. This is also given by the application of the economic principle, or here the principle of "equimarginal addition." The necessary condition for the optimum is reached when a dollar of expenditure yields the same return, in addition to social utility, in each line. This allocation is independently determinate only for fixed values of all the tax variables. For each change in the apportionment of the tax load, a new optimum allocation of expenditure must be found.

Little more can be included in the framework of the organismic theory. Vague and general terms, such as "social utility" and "social welfare," are of little use in the discussion of policy problems. The theoretical steps in the maximizing of social utility offer little or no direct guidance to governmental fiscal authorities. As was mentioned earlier, this approach has been utilized largely in the theory of budgeting. Here it can provide a primary frame of reference, within which issues may be discussed, policies formulated, and decisions reached. Even in this limited usage, however, the functional interdependence of the whole fiscal system must not be overlooked.

zation of incomes as a result of its application. This implication arises only when individuals are considered as the basic fiscal entities and when some assumption is made about the identities of individuals as pleasure machines.

III

The focus is completely shifted in the individualistic theory. The individual replaces the state as the basic structural unit. The state has its origin in, and depends for its continuance upon, the desires of individuals to fulfil a certain portion of their wants collectively. The state has no ends other than those of its individual members and is not a separate decision-making unit. State decisions are, in the final analysis, the collective decisions of individuals.

The income of the state represents payment made by individuals out of their economic resources in exchange for services provided. In the provision of these services the state is, in most cases, in a perfectly monopolistic position. However, it does not seek to maximize net revenue. Services are offered at cost. The supply curve of public services is an average-cost curve, not a marginal-cost curve. This applies, of course, only to the aggregate of all public services considered in the abstract as some sort of homogeneous magnitude. In no way does it imply that each particular service is "sold" to individuals at the average cost of provision. In the social-service state many are offered free; others at marginal-cost prices. In very few cases (the particular services provided in return for fees being the major exception) is there much connection between payments made to government by individuals and the special benefits enjoyed. But, when the aggregate of all public services is considered, the attempt is made to cover total cost. In effect, if public services are categorized, the "losses" to the government resulting from the provision of some must be balanced by "profits" accruing from the provision of others.

The extent and range of public services are determined by the collective willing-

ness of individuals to purchase them. Services will be extended as long as the aggregate benefits are held to exceed the costs. For the total of all public services, aggregate benefits should approximately equal total costs in terms of sacrificed alternatives. Ideally, the fiscal process represents a *quid pro quo* transaction between the government and all individuals collectively considered.[5] The benefit principle must be applicable in this sense.

Each individual is subjected to some fiscal pressure; his economic resources are reduced by the amount of tax that he bears. His real income is increased by the benefits that he receives from government services. The allocation of total tax load among individuals must be combined with the distribution of benefits from publicly provided services in any complete theoretical framework. However, in this approach the imputation of specific benefits to individuals has been almost entirely glossed over in the orthodox literature. Overwhelming attention has been devoted to the allocation of the tax burden.

The practical omission of the benefit side can be attributed in part to the erroneous foundations of the benefit theory of taxation. In the legitimate rejection of the benefit principle as the universal norm for the distribution of the tax burden, too much was thrown out. This principle, widely accepted in the seventeenth and eighteenth centuries[6] and reintroduced in a more sophisticated form in the 1880's,[7] is based upon the premise that there should exist a *quid pro quo* fiscal relationship between the individual and government. The principle has been overthrown for two basic reasons. First, there appears no precise manner of imputing shares of the aggregate common benefit from public services to specific individuals. A significant portion of public funds is expended for the "general welfare," not to benefit particular persons or groups. Second, it is recognized that the underlying *quid pro quo* ideal is not at all acceptable in the modern state.[8]

But these two basic objections to the benefit principle are of fundamentally different natures. The first is a technical or an administrative difficulty, which prevents the principle from having direct applicability to policy. The second rests upon a rejection of the ethical premise that there should be an individual *quid pro quo*. If we accept the *quid pro quo* ideal, the benefit principle is correct in the abstract; and the problem of individual imputation of benefits is a technical, not a theoretical, problem. Viti de Marco recognized this and therefore accepted as a working hypothesis the return of benefits roughly in proportion to incomes. Thus, adopting the ethical standard of the benefit principle, he justified a system of proportional taxation. In such a theoretical framework the apportionment of the tax load is directly dependent upon the benefit hypothesis formu-

[4] Cf. Edward D. Allen and O. H. Brownlee, *Economics of Public Finance* (New York: Prentice-Hall Book Co., Inc., 1947), p. 19.

[5] The "pure" theory should be formulated on the assumption of stability in the economic system and thus balance between the two sides of the fiscal account. "Functional" finance precepts can be fitted into the framework at a second theoretical level.

[6] For an excellent short history of the early benefit theories see Erik Lindahl, *Die Gerechtigkeit der Besteuerung* (Lund: Gleerupska Universitets-Bokhandeln, 1919), pp. 118 ff.

[7] In the works of Pantaleoni, Sax, and Viti de Marco.

[8] Wagner clearly pointed out that, once the government began to provide social services, the benefit theory became completely inapplicable (Adolph Wagner, *Finanzwissenschaft* [Leipzig: C. F. Winter, 1890], II, 431-42).

lated. If benefits were assumed to accrue equally per head, Viti de Marco would have held a system of poll taxation to be appropriate.[9]

The rejection of the benefit theory of taxation should rest not upon the difficulty of individual isolation of specific benefits but upon the unacceptability of the ethical ideal of the individual *quid pro quo*. Once this commutative fiscal relationship of the individual with government is thrown out as the norm, is there any value to be gained from attempts at surmounting the technical problem of individual-benefit imputation? In orthodox fiscal theory the implied answer to this question has been "No," with the result that benefit considerations have been seriously neglected. The whole expenditure side of the fiscal account has been given little attention. In the individualistic approach, both the total amount of public expenditure and its allocation among uses have been assumed to be fixed outside the pale of fiscal theory.

Fiscal analysis has proceeded as if all taxes were net subtractions from social income, never to be returned. J. B. Say's dictum that "the value paid to government by the taxpayer is given without equivalent or return"[10] has been implicitly accepted, although lip service has been paid to its inherent fallacy. It is evident that such a limitation leaves the body of theory incomplete and inadequate. If benefits from public services accrue to individuals as a group (and this is impossible to deny), it follows that specific benefits are received by particular individuals, regardless of the technical difficulty of dividing the common benefit among them. Aggregate benefits must be in the nature of a quantitative magnitude and thus subject to conceptual divisibility. Benefits simply cannot be forgotten. Wicksell very clearly pointed out that the procedure of leaving out the benefit side amounts to concluding that each individual actually gets no benefit whatsoever from the government services provided him.[11] And, since the addition of any number of zeroes yields a zero result, the aggregate benefit must also be zero if such an omissive assumption is made.[12] On this basis no public expenditure is theoretically justified. Furthermore, if no individuals in the social unit receive any benefit, it is apparent that no funds will be granted for the support of government. No tax bills could ever be passed by an elected representative assembly.

Only if individual shares of the aggregate benefit from public services are held to be roughly equal can the concentration of analysis on the allocation of tax burden alone be theoretically justified. It seems likely that this is the assumption made by some writers.[13] If

[9] Thus Graham says: "Since the value of most governmental services to the individual members of a community cannot be accurately assessed, nor can one take or leave them as he will, it could with some reason be contended that the ideal principle of allocation of the burden of taxation is neither progression nor proportionality but uniformity, that is, that all members of the community should pay the same amount" (Frank D. Graham, *Social Goals and Economic Institutions* [Princeton: Princeton University Press, 1942], p. 234).

[10] *A Treatise on Political Economy*, trans. C. R. Prinsep (Philadelphia: J. B. Lippincott & Co., 1855), p. 413.

[11] Knut Wicksell, *Finanztheorie Untersuchungen* (Jena: Gustav Fischer, 1896), p. 82.

[12] *Ibid.*: "Ist jener Nutzen für die einzelnen Mitglieder der Gesamtheit gleich Null, so wird auch der Gesamtnutzen nicht von Null verschieden sein können."

[13] For example, compare the following statements by J. S. Mill and Bastable:

"If a person or class of persons receives so small a share of the benefit as makes it necessary to raise the question, there is something else than taxation which is amiss, and the thing to be done is to remedy the defect, instead of recognising it and making it a ground for demanding less taxes" (J. S. Mill,

phrases such as "chargeable against the whole community"[14] and "throughout the population"[15] can be interpreted as containing the implication that the benefits from most public services are shared equally among all citizens, then the approach has been internally consistent. If individual shares in the common benefit are considered equivalent, the real problems in fiscal theory are limited to the tax side, since equals would cancel equals when benefits are included in comparing the fiscal positions of individuals.

This necessary condition for the practical omission of benefit considerations should be clearly stated and its applicability empirically tested in so far as is possible. The dependence of the validity of the orthodox approach upon this rather narrow assumption was recognized by Kaizl and by Viti de Marco.[16] But a general theoretical framework should not be so limited, even if this condition is made explicit.

The difficult problem of individual-benefit imputation must be squarely faced. It is impossible to speak of the "burden of taxation" without considering, at the same time, the benefits from expenditure made out of such taxation.[17] Even the setting-up of untested explicit hypotheses concerning benefit accrual is preferable to omission altogether.[18] The most realistic hypothesis might well be that of equal per head sharing,[19] but alternative ones should be considered and tested if possible.

The final economic position of the individual after his relationship with the "fisc" can be expressed in the form of a balance between the two sides of the fiscal account. (If we accept the *quid pro quo* premise, this balance will always be zero.) This balance can be called the "fiscal residuum."[20] If an individual's tax burden exceeds the value of benefits received from government services, he will have a positive residuum. He will pay a net tax. If the value of the share of public benefits which he enjoys exceeds the value of the contributions which he makes to government, the residuum will be negative. The individual will receive a net benefit. Only by a comparison of the residuums of individuals can the total effects of a fiscal system be

Principles of Political Economy [Boston: C. Little & James Brown, 1848], II, 354).

"But from the difficulty of discrimination it seems better to adhere to the genera rule of distributing taxation without direct reference to the results of expenditure on different classes. Injustices of this kind ought to be corrected not by the redistribution of taxation but by alterations of outlay" (C. F. Bastable, *Public Finance* [2d ed.; London: Macmillan & Co., 1895], p. 312).

[14] Henry C. Simons, *Personal Income Taxation* (Chicago: University of Chicago Press, 1938), p. 31.

[15] Allen and Brownlee, *op. cit.*, p. 192.

[16] "... das fundamentale Corollar der heutigen Auffassung der Steuergerechtigkeit ... dass nicht nur die Last der Steuer gerecht vertheilt werde, sondern dass auch die Vortheile und Emolumente der öffentlichen Institutionen gleich vertheilt werden, dass sie Allen gleich zugänglich seien" (Kaizl, *Finanswissenschaft*, II, 200, cited by Lindahl, *op. cit.*, p. 133).

"Thus one may abstract from the service of providing public security, if one assumes, and if the assumption corresponds to the facts, that the amount of security provided is equal for all productive enterprises that exchange their products. Thus, it is as if one were to cancel a common term in two terms of an equation" (Viti de Marco, *op. cit.*, p. 52).

[17] Tibor Barna, *Redistribution of Income through Public Finance in 1937* (London: Oxford University Press, 1945), p. 3.

[18] *Ibid.*

[19] The use of this hypothesis has been found to yield important results in some areas. I have applied this to the theory of intergovernmental fiscal adjustment in a federal state and have been able to work out a determinate system of transfers without reference to particular tax burdens or particular service standards.

For another example of application to a similar problem see J. R. and U. K. Hicks, *Standards of Local Expenditure* (London: Cambridge University Press, 1943), p. 3.

[20] The concept of a fiscal balance or residuum was utilized by Wicksell in a different sense (*op. cit.*, p. 81).

analyzed and evaluated. Tax-burden comparisons alone are likely to yield quite different and perhaps misleading conclusions.[21]

This approach enables a general classification of fiscal systems to be made, comprising three major groups. First, those systems which tend to increase the inequality in the distribution of real income among individuals can be classified as "aggravative."[22] This type of system would be indicated if low-income individuals showed positive residuums, i.e., taxes in excess of benefits, while the high-income receivers showed negative residuums. Some systems of the sixteenth and seventeenth centuries which collected revenues from large elements of the population in the main for the support of the royal household and the nobility would perhaps be characteristic.

Second, those fiscal systems which tend to return to all individuals approximately the equivalent of their contributions and thus to have no net effect on the prevailing distribution of real income can be classified as "status quo" systems. This type would be indicated if a calculation of the fiscal residuums for individuals in all income brackets yielded roughly zero results. This type is perhaps best represented in the fiscal systems of the early nineteenth century, when a large share of revenue was collected from levies on property and a major portion was expended in the provision of protection, internal and external.

Third, those fiscal systems which tend to redress the prevailing distribution of real income toward more equality can be classified as "equalitarian" or "redistributive." If people in the low-income groups receive more in benefits than they pay in taxes and the upper-income groups contribute more than they receive in benefits, this type of system is indicated. This is, of course, the type which is characteristic of the modern state. A large share of revenue is derived from the proceeds of progressive income taxes, and a major portion of expenditures in peacetime is devoted to the provision of social services.

As was stated earlier, regardless of the kind of fiscal system, there should exist the *quid pro quo* relationship between government and all individuals taken together. This is represented by a balancing of the net taxes paid by certain individuals against the net benefits received by others in the first and third classifications. Only in the second type of system does the collective equalization of benefits and taxes imply that each individual receives in benefits the approximate equivalent of contribution made.

This simple classification represents nothing new.[23] It seems essential, however, that it be employed as the primary frame of reference within which more specific problems may be placed. One should bear in mind that the classification refers to fiscal systems, not to tax

[21] The comparative study of the burdens of state and local taxes in New York and Illinois for 1936, published by the Twentieth Century Fund in *Studies in Current Tax Problems* (New York, 1937), can be used as an example. On the basis of tax-load comparisons alone, the tax load was found to be heavier in New York than in Illinois on eight out of the ten hypothetical families studied (p. 34).

By making the assumption of benefit accrual equally per head, and including New York and Illinois state and local expenditures in addition to the computed tax-load figures, it was found that the results were significantly changed. Where only two New York families out of ten were found to be in advantageous positions when tax burdens alone were considered, seven New York families became favored in fiscal treatment when the benefit side was included.

[22] The use of the word "aggravative" here, of course, indicates the egalitarian "bias" of the writer.

[23] Cf. W. J. Shultz and C. L. Harriss, *American Public Finance* (5th ed.; New York: Prentice-Hall Book Co., Inc., 1949), p. 104.

systems. Tax systems have been traditionally classified as regressive, proportional, and progressive, based upon the ratio of tax burden to income at different income levels. These terms are useful in describing the nature of a tax system, but they do not describe the fiscal system, as would appear to be the case in ordinary usage.

Progressive taxation has been justified because it leads to a more equal distribution of income among individuals. Standing alone, the statement that progressive taxation does redistribute real income is not true. It can be true only on the basis of certain assumptions about the other half of the fiscal system. If individual shares in the common benefit from public expenditures are all equal or approximately so, then progressive taxation will lead to a more equal distribution. But so will proportional taxation on the same basis.[24] Without considering the imputation of individual benefits, all that may be stated categorically is that progressive taxation will produce a more equal distribution of income than would proportional or regressive taxation. One cannot say that it will produce a redistribution; it may or may not.[25]

If a major share of governmental expenditures were allocated to provide protection to the property rights of the wealthy classes, then even a progressive tax system might not prevent the fiscal system from increasing real income inequality. On the other hand, if benefits were wholly in the nature of social services, poor relief, unemployment compensation, etc., then a *regressive* tax system might well be a part of a *redistributive* fiscal system.[26] Conversely, public expenditures made for the benefit of low-income groups may be financed by taxes on those same or even lower-income groups.[27] Certainly, if a significant tax increase or change in the structure of the tax system of any kind is proposed, the ultimate manner in which the proceeds are to be expended should, in part, determine the nature of the change. The same amount of redistribution may be as well accomplished by the levy of a sales tax to provide expanded social services as by an increase in the higher-bracket income-tax rates to finance additional defense expenditure. Since redistribution is only one goal of responsible social policy in the modern state, fiscal policy should reflect such possible alternatives.

The ratio of tax burden to income at various income levels should be separated from any underlying ideas concerning final distributive effects. Owing to the connotation which has caused them to be so sharply categorized with such distributive implications, perhaps it would be better if the terms "regression," "pro-

[24] The statement that progressive taxation will redistribute incomes but that proportional taxation will not implies that benefits are returned to individuals in proportion to incomes and wealth.

[25] The emphasis upon the redistribution effects of the tax side alone is indicated in the following quotation for the *Annual Economic Report* submitted to the President by the Council of Economic Advisers in January, 1949: "The federal personal income tax has reduced somewhat the concentration of income. In 1947, for example, the lowest three-fifths of families received 29 per cent of total money income before tax compared to 31 per cent after tax, while the share of the upper one-fifth was reduced by taxes from 48 to 46" (reprinted in *United States News and World Report*, January 14, 1949, p. 72).

See also R. A. Musgrave and Tun Thin, "Income Tax Progression, 1929–48," *Journal of Political Economy*, LVI (1948), 498–514.

[26] In 1947 a state-wide general sales tax was imposed in Tennessee, with most of the proceeds earmarked for expenditure in the provision of educational services. It can plausibly be argued that the collection of the tax and the expenditure of the proceeds taken together are redistributive.

[27] The British case seems applicable here. Should not the "redistributive" effects of the food subsidies be carefully scrutinized, considering the enormous levies on tobacco and liquor?

portion," and "progression" were discarded.

The theoretical framework in the individualistic approach does not include the specification of a single fiscal system to be adopted by the society. The ends to be served by the fiscal system are determined by political decisions. The framework does, however, enable the fiscal specialist to indicate the alternate distributions of tax burdens and public expenditures which will yield the desired results. If, for example, the society desires a fiscal system which will not affect the prevailing distribution of real incomes, any number of fiscal structures, tax-burden and expenditure allocations, can be formulated which will approximate it. Further, if this status quo ideal, plus an allocation of public expenditures, is given, the single most appropriate tax structure can be outlined.

A similar approach is suggested for other than status quo norms. The redress of the prevailing income distribution toward greater equality has been accepted as one of the fundamental purposes of the fiscal system in the modern state. The fiscal scientist can provide policy-makers with practical guides to action in several ways. If the desired degree of redistribution is known, the alternate pairs of tax-burden and public-expenditure allocations which will yield this result can be indicated. If this, plus the existing public expenditure pattern, is known, it is possible to set up the tax system, assuming in this case that attainment is possible under the given conditions. On the other side, given the degree of redistribution and the apportionment of the tax load, the expenditure pattern can be established.

Society, however, does not normally make concrete decisions concerning the amount of redistribution desired. Rather, it determines an allocation of expenditures among uses and a distribution of the tax load which will, when combined, cause the fiscal system to be redistributive. The role of the fiscal scientist qua scientist in this situation is clear. With these properties he can determine roughly the amount of redistribution of real incomes actually accomplished through the fiscal process.[28] He is then able to indicate alternative tax and expenditure allocations which would yield approximately equivalent redistribution results, some of which might result in significantly different effects upon the economy. It is perhaps in the area of estimating the amount of redistribution carried out by the fiscal system that the most productive empirical work in the whole field of government finance can now be carried on. The traditional difficulty encountered in the attempt to impute specific shares of the benefits from public services to individuals, even upon the recognition of the communal nature of the aggregate benefit, should not be deterrent. It will not be, once it is fully comprehended that the benefit side cannot be left out and any sort of generalized theoretical framework set up. It must be recognized that the omission yields results equivalent to those based upon even more arbitrary assumptions than the heroic ones admittedly required concerning the incidence of public expenditures. With the expansion of government activity toward the provision of social services, and services to particular economic or social groups, the problem should present less of a dilemma. For example, the individual benefits from farm price supports can be more readily estimated than those from defense expenditures.

[28] This is the approach taken in the path-breaking work of Barna (*op. cit.*)

IV

In both the organismic and the individualistic approaches to fiscal theory, the paramount need is that the interdependence of the two sides of the fiscal process be clearly understood. Both approaches require parallel consideration to be given to the determination of the expenditure allocation and the apportionment of tax burden. In neither theory can either side be analyzed in isolation.

The organismic framework gives a much more complete normative behavior pattern for the fiscal authority. Since the government is the basic entity, the fiscal theory reduces to a statement of an applied maximization problem. The major obstacles lie in the attempts at translation of the theoretical guides to action into a realistic approach to practical policy. It becomes extremely arduous, if not impossible, to fill in the theoretical framework with empirical content.

In the individualistic approach the government represents only the collective will of individuals and cannot be considered the originator of action in an abstract sense. The fisc cannot be assumed to maximize anything. The fiscal system exists as one channel through which certain collective desires may be accomplished. The content of theory becomes the setting-up of a structural framework to enable the results of policies to be evaluated.

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LIQUIDITY AND A NATIONAL BALANCE SHEET

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THIS essay is an attempt to analyze the factors determining "liquidity position" and to show the need for a "nation's economic balance sheet" to supplement "the nation's economic budget." Greater precision in thinking about liquid assets and illiquid obligations may enable us to formulate and test improved hypotheses concerning changes in these balance-sheet items and in the level of aggregate demand. Before examining particular types of assets and debts, I shall summarize (1) present views on the relationship between liquidity and spending and (2) the development of the concept of liquidity position.

I. CURRENT HYPOTHESES ABOUT EFFECTS OF CHANGES IN LIQUIDITY ON VOLUME OF SPENDING

Practically all schools of thought believe that fluctuations in liquidity influence the level of aggregate expenditure. An increase in the money stock, for example, may induce exchanges of cash for less liquid assets of all varieties—for claims, raising their price and lowering their yield or "the" interest rate; for capital goods, pushing investment to a correspondingly lower position on the marginal efficiency of capital schedule; and for consumers' goods, raising the consumption function and lowering the marginal rate of substitution of consumers' goods for other assets.

Even an old-fashioned Keynesian like Roy Harrod goes along with this when he writes (regarding the postwar inflationary pressure in Britain): ". . . if in the existing state of affairs there is a tendency for people to save less or to expend old savings, that is due to the low rate of interest and has nothing to do with the large volume of liquid assets, which is merely the mechanism by which that low rate has been brought about."[2] Despite this rather odd way of putting it, he is declaring that new balances will be allocated among different uses so as to equalize the marginal gain obtainable in each direction.[3]

Moreover, Keynesian converts, like A. G. Hart, who emphasize the imperfections of the capital market, would insist that an increase in the stock of liquid assets may increase investment without depressing *observable* interest rates.[4] At least two explanations are possible. One is capital rationing. In a single market for a homogeneous commodity, sellers do not prefer one customer to another, and price tends to move so as to equate quantity demanded and quantity offered. In the loan market, however, sellers (i.e., lenders) classify their customers according to the risk of losing principal and interest payments, and no single interest rate moves so as to equate the quantities offered and demanded. If customers fell

[1] I am especially indebted to Professors Milton Friedman and Lloyd W. Mints for helpful criticisms.

[2] Roy Harrod, *Are These Hardships Necessary?* (London: Rupert Hart-Davis, 1947), p. 128.

[3] Many would urge, of course, that the marginal gain in certain uses will drop rapidly, so that very little new cash will be devoted to these uses.

[4] A. G. Hart, "Assets, Liquidity, and Investment," *American Economic Review, Papers and Proceedings*, XXXIX (May, 1949), 171–81.

into discrete compartments or risk-categories, this would simply mean that several loan markets and interest rates existed. But even within a single broad risk-category, *each* borrower presents a different degree of risk; and a loan to one customer is a different commodity from a loan to another. There can be no smooth supply schedule of loans, no offer of various volumes at a series of interest rates to each category of borrowers. Hence not only must there be many different markets for loans and many interest rates, but, at each resulting rate, some rationing is inevitable. Since there can hardly be as many interest rates as there are degrees of risk, preference among customers must help interest rates allocate funds; for a price to equate quantity demanded and offered in a market for full titles to existing goods is conceivable; but in a market for contracts concerning the future, in an uncertain world, differentiation among customers and rationing at each price seem unavoidable.[5] A shrinkage in the volume of liquid assets, therefore, may not produce lower supply schedules of loanable funds in all the loan markets (separated according to the degree of risk), so that smaller volumes of loans are negotiated in each market at higher interest rates. Rather, the shrinkage in liquid assets may produce higher debt-asset ratios, fewer eligible and willing borrowers all along the line, and smaller volumes of loans negotiated at each old interest rate, with the higher-risk borrowers excluded entirely. Conversely, an increased volume of liquid assets may produce lower debt-asset ratios, more eligible borrowers, and a greater volume of loans negotiated at the previously existing pattern of interest rates. Investment may be stimulated without any decline in observable interest rates.[6]

A second explanation can be offered without resort to the concept of "credit rationing," i.e., without encroaching upon the allocative role of interest rates. Fewer liquid assets may lower *both* demand schedules for, and supply schedules of, loanable funds, since less favorable asset-debt ratios in an uncertain world involve extra risk for both entrepreneurs and lenders. The result could be a restricted volume of loans without a rise in interest rates. The supply of loans to high-risk borrowers could dwindle to zero, as they move into even higher-risk categories, where supply and demand schedules never intersect; there would be no transactions and no recorded price (as in the market for rubber skillets). On the other hand, supply schedules of loans to low-risk customers (like the government) could increase so that such loans continued to take place at even lower interest rates. Even if interest rates do perform the allocative task, a decreased (increased) volume of liquid assets can shift demand and supply schedules for loans so as to discourage (encourage) investment without raising (depressing) observable interest rates.[7]

[5] See *ibid.* for comments on the possibility of having pure competition in the capital market.

[6] *Cf.* M. W. Reder's "Discussion" in "The Theories of J. M. Keynes," *American Economic Review*, XXXVIII (May, 1948), 297: "To my way of thinking, it is quite improper to eliminate availability of funds as a determinant of the rate of investment. However, the model that seems to me correct is neither Keynesian nor neo-classical.

"Such a model would be based squarely upon the fact of capital rationing. Each firm would have its own sources of capital, the supply of which would not be closely related to the rate of interest it must pay. . . . It would be possible to have low open-market rates coupled with a shortage of available funds for borrowers whose securities did not qualify for the open market."

[7] These conclusions do not depend upon changing expectations and resulting changes in the demand for loans or in the marginal efficiency of capital.

In fact, the complex of observable interest rates may appear to rise (fall) as funds become more (less) "available."

To summarize this introductory section, Keynesians and non-Keynesians alike agree that changes in liquidity positions are likely to influence the level of consumption and investment, though how and how much are still controversial matters.[8]

II. THE DEVELOPMENT OF THE CONCEPT OF LIQUIDITY POSITION

Individual businessmen certainly look at more assets than cash and also at the other side of the balance sheet before coming to any conclusions about their liquidity position. A banking firm can hardly base its lending decisions solely upon an examination of cash on hand. Yet, in elaborating the quantity theory of money, economists tended to obscure the other balance-sheet items and to focus attention on the influence of one particular asset—money—which had to be defined arbitrarily. Some—Henry Thornton was one of the earliest, and Henry Simons one of the most persuasive—sought further in the balance sheet for influences on the level of spending and emphasized that liquid assets other than those defined as money were near-moneys or money substitutes.[9]

Then, in the 1930's, Irving Fisher developed the debt-deflation theory of depressions, stressing private debts and debt burden as partial explanations of fluctuations in aggregate spending.[10] Henry Simons improved the hypothesis, particularly with respect to short-term debts and the liquidation scramble.[11] By and large, however, Fisher and Simons tended to discuss the implications of privately held liquid assets apart from the implications of debts privately owed. They made little attempt to consider the two sides of the balance sheet simultaneously in order to formulate an integrated hypothesis.

Some investigators have attempted to observe the whole balance sheet rather than just certain components. A. G. Hart, in *Debts and Recovery*, was concerned throughout with both assets and liabilities in relationship to each other and to the volume of spending.[12] J. R. Hicks, in 1935, urged that monetary theory ought to be developed by regarding ". . . every individual in the community as being, on a small scale, a bank" and that ". . . the same forces which work through the assets side work through the liabilities side in much the

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[8] In order to analyze the effect of balance-sheet items apart from that of income flows, we have to confine the discussion to existing, rather than expected, liquidity positions. A theory of expected liquidity positions or an entire theory of assets would swallow up all theory, inevitably embracing not only expected incomes and outlays but expectations of all sorts.

[9] Henry Thornton, *An Enquiry into the Nature and Effects of the Paper Credit of Great Britain* (London, 1802), pp. 39–43; Henry Simons, "Rules versus Authorities in Monetary Policy," *Journal of Political Economy*, XLIV (February, 1936), 5–14, reprinted in *Economic Policy for a Free Society* (Chicago: University of Chicago Press, 1948), pp. 164–72.

Recently, numerous writers have discussed or investigated liquid assets other than money. See, e.g., Hart, "Postwar Effects To Be Expected from Wartime Liquid Accumulations," *American Economic Review*, XXXV (May, 1945), 341–51; U.S. Department of Agriculture, Bureau of Agricultural Economics, Division of Program Surveys, *National Survey of Liquid Asset Holdings, Spending, and Saving: A Survey Conducted for the Board of Governors of the Federal Reserve System* (Washington, 1946); and further analyses reported at intervals in the *Federal Reserve Bulletin*.

[10] *Booms and Depressions* (New York: Adelphi Co., 1932).

[11] *Op. cit.*

[12] Hart, *Debts and Recovery* (New York: Twentieth Century Fund, 1938).

same way. But this certainly requires further exploration."[13]

Homer Jones has tried to investigate the relationship between the balance sheet and the volume of investment spending.[14] He has found that individual savers have a remarkably strong preference for holding fixed claims (e.g., time deposits, insurance policies, bonds).[15] Furthermore, financial institutions are under legal and moral pressure to hold predominantly fixed claims. Therefore, Jones decided that as the debt-equity ratio rises, there is less "room" for investment because investors refuse to lend and businessmen refuse to borrow when debt-asset ratios are unfavorable. Bankers, accountants, businessmen, farmers, and even individual households keep an eye on debt-asset ratios because they realize that this is an uncertain world. Since the 1945–46 debt-equity ratio was exceptionally low (i.e., the national balance sheet had become heavy with assets compared to private debts), Jones concluded that investment prospects were excellent but that the rise in the debt ratio as investment in fixed claims soared might lead later to a drying-up of investment. We should notice that, as the debt ratio rises, both the demand schedule for and—if it makes sense to speak of them—the supply schedules of loanable funds are lowered without necessarily causing a rise in interest rates, particularly those paid by low-risk borrowers. Perhaps it should be observed also that inflation drives the debt-equity ratio down, encouraging further expansionary investment (at least until prices and incomes rise less rapidly than debt) and that deflation drives the debt-equity ratio up, discouraging investment outlays still further (at least until private debt falls more rapidly than prices and incomes). *During fluctuations*, of course, other influences, notably price and income expectations, may play a dominant role.[16]

III. ANALYSIS OF FACTORS DETERMINING LIQUIDITY POSITION

First, it is necessary to define several terms. "Liquidity" will be used here to mean merely "moneyness." Usually, an asset's liquidity is described to include the probabilities of getting various fractions of the going price plus the time period necessary to liquidate the asset. Perhaps another element should be added—the time period over which the other characteristics are expected to endure.[17] Since these components cannot

[13] "A Suggestion for Simplifying the Theory of Money," *Economica*, II (new ser.; February, 1935), 12, and 13, n. 2.

[14] "Investment Prospects," *Journal of Finance*, II (April, 1947), 15–33.

[15] "The Flow of Savings. I," *ibid.*, III (October, 1948), 6–7.

[16] These attempts to include both assets and liabilities as determinants of liquidity positions and, through them, of outlays appear to be only a beginning. It may be useful, for example, to break down the aggregates and examine the relationship of various types of claims and debts to liquidity positions.

[17] Many definitions apparently ignore this aspect of liquidity, for instance: "Roughly speaking, the longer the useful or consumable life of the article, the less liquid it is likely to be" (A. A. Berle, Jr., and V. J. Pedersen, *Liquid Claims and National Wealth: An Exploratory Study in the Theory of Liquidity* [New York: Macmillan Co., 1934], p. 42). But surely it is not an apt definition of liquidity that makes such a statement valid. Money, possessing 100 per cent liquidity, is expected to last indefinitely; any asset which depreciates rapidly, whether from use or disuse, does not have much "moneyness."

C. C. Brown excluded from liquidity the ability to exchange for a particular price, which, he said, involved appreciation or depreciation, a separate and confusing quality possessed by assets (*Liquidity and Instability* [New York: Columbia University Press, 1940], pp. 3, 7). It is true that appreciation or depreciation in real value is a characteristic of both liquid and illiquid assets and is hence something

be measured, there is little to be gained by breaking the notion down. Perhaps it is sufficient to say that the more nearly we regard an asset as substitutable for money, or the more it partakes of the same attractions possessed by money-holdings, the more liquidity the asset has.

"Illiquidity," on the other hand, will be used to mean the opposite of moneyness. In contrast with the liquidity of claims—the ability to make lower cash balances acceptable—is the "illiquidity" of debts, or the capacity of maturing debts to require the holding of cash balances. That debtors to banks do, in fact, maintain higher balances than they would otherwise keep was demonstrated by the well-known analysis of C. A. Phillips.[18] After a borrower gets a loan, there is some lag before all the proceeds are checked out. Indeed, the proceeds may never be completely drawn out, especially since many banks require the maintenance of minimum balances. As the maturity date approaches, cash is gradually accumulated to retire the debt. On the average, Phillips estimated that 20 per cent of a bank's created deposits remained with the original bank (though this figure would vary, depending upon a great many factors). This amounts to saying that, on the average, debtors to banks maintain higher cash balances, to that extent, on account of their obligations. This "illiquidity" of debt depends upon lags, the probability of having to pay the face amount without renewal or refinancing, and the time available before repayment. To repeat, the illiquidity of debts is the extent to which they act as a reduction of the moneyness of balance sheets—the extent to which they require the maintenance or acquisition of cash balances.[19]

The liquidity position of a person or spending unit means the "current" position, and "is determined by the *liquidity characteristics* and *money amount* of assets owned, relative to the *maturity schedule* and *money amount* of debts owed."[20]

The liquidity position of the entire economy is the aggregate or combined liquidity positions of private individuals, nonfinancial businesses, financial institutions, and governments. The analysis here, however, will be confined to the liquidity position of individuals and nonfinancial businesses.[21] Only balance-sheet items as they would appear on their books will be considered. Assets and obligations on the books of lending institutions and government units are important to aggregate spending but can best be examined separately.[22] I am concerned with those balance-sheet items which directly affect the consumption and investment outlays of businesses and individuals. Although it is well to remember the similarities between banks and

apart from liquidity. Nevertheless, the tendency not to depreciate or appreciate in nominal value is a trait of moneyness.

[18] *Bank Credit* (New York: Macmillan Co., 1926), chap. iii.

[19] Hart uses the terms "negative liquidity" and "illiquidity" in this same sense in *Money, Debt, and Economic Activity* (New York: Prentice-Hall Book Co., Inc., 1948), pp. 75–76, 133–35.

[20] Brown, *op. cit.*, p. 151.

[21] Admittedly, the distinction between financial and nonfinancial businesses must be arbitrary, but it is sufficient to include in the former all banks, life insurance companies, savings and loan associations, etc., that accept "deposits" or owe fixed obligations and relend on a large scale.

[22] The federal government's liquidity position (or the size of the public debt) affects legislation and appropriations and may exercise a depressing, or other, psychological influence on private spending. The liquidity position of financial institutions affects their willingness to lend, the illiquidity of private debt to the banks, the ability to borrow from the banks, and hence the liquidity position of nonfinancial businesses and individuals.

other businesses,[23] separate treatment seems justified by the differences—notably that (1) banks' "spending" policy, for the most part, affects the nonfinancial sector's liquidity position rather than directly affecting consumption and investment; (2) banks carry much larger cash reserves (i.e., are probably influenced to a much greater degree by liquidity position) than other businesses; and (3) banks' liquidity positions are directly regulated to a considerable extent by monetary authorities.

In the following sections I shall examine several classes of assets in conjunction with the liabilities, if any, which correspond to them. Claims are matched by different types of debts. Unless one takes into consideration the impact on the debtor's cash-balance requirements, or liquidity position, as well as on the creditor's, a short-term claim against private business will appear to be much the same in its effects as a short-term claim against the government. Yet the net effects are clearly not the same. Three ways in which liquidity position may be affected will form the headings under which these balance-sheet items will be discussed: (A) changing volume of assets and debts, (B) changing liquidity of assets and illiquidity of debts, and (C) changing real value of claims and real burden of debt as the price level fluctuates.

A. CHANGING VOLUME OF ASSETS AND DEBTS

Currency, demand deposits, near-moneys, and equity titles not matched by private debt.—It is impossible to say, once and for all, precisely what should be included in such categories as money and near-moneys (i.e., acceptable substitutes, to variable extents, for cash balances).[24] I include currency and demand deposits under money, since they are usually acceptable for either cash-balance or exchange purposes, and I arbitrarily define near-moneys as claims, other than demand deposits, against financial institutions or the federal government. Thus they include time deposits, cash-surrender values of life insurance policies,[25] repurchasable shares in savings and loan associations, travelers' checks, and privately held obligations of the federal government, such as postal savings, short-term bills, and all redeemable and marketable securities.

Currency, demand deposits, near-moneys, and "other fixed claims" (referred to below) have fixed face values, while equity titles are assets having no fixed face amounts. Fixed claims can, of course, have variable exchange value; they can become worthless or be transformed through foreclosure or agreement into equity titles. There are a few hybrid types, but most assets fall logically into one of the two classes. So much for the way in which these terms are used here.

Pigou has explained that one effect of

[23] As Hicks observed: ". . . the cash reserve a bank keeps against its liabilities is simply a special case of the holding of money against uncertain future expenditures, which is practiced to some extent by all businesses, and by many private individuals as well" (*Value and Capital* [Oxford: Clarendon Press, 1946], p. 241).

[24] For a good statement of the problems involved see M. Bronfenbrenner, "Some Fundamentals in Liquidity Theory," *Quarterly Journal of Economics*, LIX (May, 1945), 405, 413.

[25] Despite the similarities between cash-surrender values or "deposits" with insurance companies and other time deposits, many persons feel that the former are seldom regarded as money substitutes. If it is true that people are extremely reluctant, by and large, to "tap" these liquid reserves, then, in effect, they are not liquid reserves and cannot substitute for cash. Nevertheless, all these claims can be (and, in an emergency, are) converted into cash; so for purposes of analyzing the theory of liquidity position, I have classed all claims against financial institutions as near-moneys.

an increased amount of money issued by the government is to increase the real value of the total volume of claims and hence to increase the real value of the community's total stock of assets.[26] This ought to make people feel better off and, since the proportion of real value in the form of liquid claims has increased, in a more liquid position. An increase (decrease) in the volume of metallic or paper money, *ceteris paribus*, is a clear-cut net addition to (deduction from) the real value of liquid assets, with no offsetting change in the real burden of private debt.[27] Likewise, if the government sells new securities to the central or commercial banks (and spends the proceeds), the volume of demand deposits not matched by any private debt expands,[28] adding to the net liquidity of the community.

Of course, other things do not normally remain the same. Currency in circulation fluctuates according to the form in which people wish to hold their liquid assets. Conversion of currency into deposits, or vice versa, alters the volume of liquid assets other than currency (and also alters the banks' liquidity position, neglected here, unless government action deliberately maintains perfect interconvertibility). In the case of demand deposits not matched by private debt, bank purchase of government bonds from individuals or businesses increases the stock of demand deposits not matched by private liabilities but simultaneously decreases the public's bond holdings. Member-bank purchase of bonds from the government simultaneously reduces the banks' liquidity (and therefore, indirectly, the public's liquidity), though it does not appear on the balance sheet of the nonfinancial public. Analyzing changes in each asset, with other things held the same, serves to point up the distinctions between different balance-sheet items; the obvious abstractions involved emphasize the necessity of observing changes throughout the balance sheets of both the nonfinancial public and the financial institutions.

Another type of liquid asset not matched by private debt is the stock of near-money claims against the federal government, their moneyness depending upon the terms and maturities. Longer-term claims—if support prices are absent or changeable—will have less liquidity than short-terms, though both will be more liquid than corporate bonds with similar maturities. The near-money claims against financial institutions, too, are unmatched by private obligations to the extent that they are offset by government securities, equities, or reserves held by the financial institutions. An increase (decrease) in the volume of all these near-moneys, *ceteris paribus*, should add to (detract from) the community's liquidity position.

Properties owned outright, also, though not claims to a definite number of dollars, must be an addition to, not a detraction from, liquidity positions. Equity titles vary a great deal in liquidity; common

[26] "The Classical Stationary State," *Economic Journal*, LIII (December, 1943), 342–51, and "Economic Progress in a Stable Environment," *Economica*, XIV (new ser.; August, 1947), 180–90.

[27] This is true only under the special circumstances existing in the United States. If currency is issued by commercial banks, the subsequent analysis of demand deposits matched by private debt is applicable.

[28] Debts of state-local governmental units, omitted here because relatively unimportant, are in an intermediate position between federal and private debt. Although local governments possess the taxing power, they do not have the authority to create money or the central government's relationship to the banking system. Moreover, we know that local debts are repudiated and that local governmental balance sheets probably influence their outlays more than in the case of the federal government.

stocks on major stock exchanges, inventories, jewelry, and some types of real estate are among the comparatively liquid properties. Ownership is not matched by an offsetting liability; hence, to the extent that these equity titles have liquidity, they make a net contribution to the community's liquidity position. In the aggregate the real value of these assets probably changes slowly, although the growth of common-stock financing expands the volume of equity titles and improved markets or rapid growth of inventories can shift the composition toward more liquid types.

All the classes of assets discussed in this section make an undiluted contribution to liquidity positions.

Demand deposits, near-moneys, and other fixed claims matched by private debt.—Expansion or contraction of the volume of these claims cannot affect the real value of the community's holdings, because the real value of the new claims is offset by the real burden of the new liabilities. Thus the Pigou line of reasoning is inapplicable. If, moreover, debtors regarded their obligations as detracting from their liquidity positions as much as the owners of claims regarded them as adding to their liquidity positions, then the creation and destruction of the claims could have no net effect on the liquidity position of the community. For two reasons, however, it is unlikely that debtors do feel impelled to increase their cash balances on account of their obligations as much as claim-holders are enabled to reduce their previous cash balances on account of the newly created demand deposits, near-moneys, or other claims: (1) In the case of new private deposits with, matched by new private borrowing from, financial institutions, the debts to the institutions are seldom due on call and are often term loans or long-term bonds, while the claims against the institutions are available on short notice (and, in the case of checking deposits, serve as a circulating medium). The more remote the maturity of the debt to the bank or insurance company and the more immediate the availability of the claim, the greater the excess of the claims' liquidity over the debts' illiquidity. (2) Even if the claims and debts have identical terms and maturities, as in the case of other fixed claims (i.e., against individuals and nonfinancial businesses), the claims are surely regarded as being more liquid than the debts are illiquid. Otherwise, there would be no point in borrowing and lending; expansion of debt would never take place if the borrower believed it necessary to raise his cash balance as much as the creditor lowered his.

In other words, for one or both of the above reasons, claims are more liquid, considered by the holders, than the corresponding debts are illiquid, considered by the debtors.[29] Expansion (contraction) of these claims, *ceteris paribus*, puts the community in a more (less) liquid position, though not to the extent suggested if the matching private debts are overlooked.

Again, it is important to remember that other things do not usually remain the same. Expansion of privately held money or near-money will occur if financial institutions purchase existing cor-

[29] Hart, *Money, Debt, and Economic Activity*, pp. 75–76, 134; Hicks, "A Suggestion for Simplifying the Theory of Money," pp. 10–12.

In the next section I state that, during deflation, claims become less liquid, while debts become more illiquid, but the debts are not likely to develop more illiquidity than the liquidity retained by the claims. That is, before the debtor maintained higher cash balances on account of the liability than the creditor would forego on account of his asset, the two would be likely to retire the loan.

porate bonds, without any change in private liabilities. In this case substitution of highly liquid checking accounts or near-moneys for less liquid long-term bonds adds to the public's (but detracts from the institutions') liquidity position. The effects are similar to, but probably stronger than, the effects of purchasing existing government securities by the banking system. Or institutions may vary their holdings of reserves or cash instead of matching depositors' claims with any kind of earning asset.[30] Debts to institutions can fluctuate without any change in claims against them (or vice versa). To put it another way, the volume of claims against institutions not matched by (i.e., in excess of) private debts to them will increase if holdings of government securities, equities, or reserves are built up at the expense of private debts to banks.

Another important qualification is that contraction is scarcely ever the result only of lenders' compelling debtors to repay at maturity or "persuading" others to exchange cash for bonds on the open market. Often debtors voluntarily repay their obligations. Their demand for cash is no greater than their desire to get rid of the debts. If the banks are as willing as ever to relend on the same terms, paying off bank loans becomes just a way of "storing" checking accounts. The volume of demand deposits privately held and of debts privately owed does decline; yet under these circumstances the resulting excess reserves must be considered as taking their place. Likewise, if insurance companies, other institutions, or individual lenders remain as anxious as before to lend funds voluntarily repaid, the community's liquidity position is not impaired. This means that the financial institutions' liquidity position, though put to one side in this discussion, is indispensable for interpretation of nonfinancial balance sheets.[31] Ability to borrow, elusive as the notion is, must somehow be recognized as a money substitute and as a part of liquidity positions.

All the classes of assets discussed in this section make a "diluted" contribution to the community's liquidity position. Demand deposits are naturally more liquid than near-moneys, and the latter are more liquid than other claims (such as trade receivables, privately held mortgages, or corporate bonds); but the argument is similar for all: expansion decreases the cash-balance needs of the holders more than it increases the cash-balance requirements of the debtors, and contraction ordinarily does the opposite.

B. CHANGING LIQUIDITY OF ASSETS AND ILLIQUIDITY OF DEBTS

Claims may possess what has been entitled "fair-weather" liquidity,[32] and

[30] See Jones, "The Optimum Rate of Investment, the Savings Institutions, and the Banks," *American Economic Review, Papers and Proceedings*, XXXVIII (May, 1948), 321–39, for evidence concerning the extent to which this has occurred.

[31] In the case of voluntary repayment to commercial banks, the "disappearance" of demand deposits (i.e., the emergence of excess reserves) is simply an alternative to a decline in velocity (though strictly passive behavior by banks would be difficult to demonstrate). Under our banking system today the repayment of loans by private borrowers may be predominantly voluntary. Friedrich Lutz's findings led him to believe that large corporations would accumulate cash during depressions and leave it idle if the banking system did not receive repayments and "destroy" checking accounts: "These comparisons suggest that in a system in which business is not indebted to the banks to any great extent—i.e., in which deposits are created or canceled through the purchase or sale of government securities—business fluctuations may reflect themselves not in an increase or decrease of bank loans and deposits but in a change in the velocity of circulation of deposits and, perhaps, in a change in ownership of deposits between consumers and enterprise" (*Corporate Cash Balances, 1914–43* [New York: National Bureau of Economic Research, 1945], p. 5).

[32] Edward C. Simmons, "The Relative Liquidity of Money and Other Things," *American Economic*

debts may possess what might be termed "foul-weather" illiquidity. During good times, when there is no mass attempt to call claims, they are extremely liquid and function as good money substitutes; debts, on the other hand, which can be renewed or refinanced easily, are not very illiquid, and little cash reserve is needed to meet obligations. When critical times arrive, however, and general liquidation is attempted, creditors doubt the moneyness of their claims, and debtors encounter increased pressure to pay and greater obstacles to refinancing. Hence claims lose their ability to substitute for cash, and debts acquire the capacity to require the accumulation of cash balances. This foul-weather illiquidity of debts is about what A. G. Hart once entitled the "mythology of maturities."[33] During prosperous times, debts have no effective maturity, because they can easily be refinanced. During bad times the maturity of obligations is very real and, in many cases, accelerated.

To include fluctuating liquidity of claims and illiquidity of debts as determinants of net liquidity positions is to introduce the by-products of changing liquidity desires. It may appear to erase all distinction between liquidity position and liquidity desire, but, in reality, I believe, it enables us to make the distinction sharper. In a community with absolutely no private debts, fair-weather liquidity and foul-weather illiquidity would not exist, though both the stock of, and the desire for, liquid assets could fluctuate. In our community, with private debts constituting much of our financial structure, fluctuations can occur in (1) the stock of liquid assets, (2) the desire for liquid assets, and (3) the liquidity of balance-sheet items as a by-product of changing liquidity desire.

Currency, demand deposits, near-moneys, and equity titles not matched by private debts.—Currency issues by either the Federal Reserve banks or the Treasury can hardly suffer from fair-weather liquidity any more, and no offsetting private debt exists to suffer from changing illiquidity. Near-money claims against the federal government, which has the power to substitute cash for its obligations, are not likely to change much in moneyness.[34] The longer-term securities will have less moneyness than will the short-terms and some degree of fair-weather liquidity, not due to doubts that the government can redeem them but due to changing "shiftability," interest rates, and uncertainty about support policy.

Demand deposits and near-money claims against financial institutions in excess of private debts to them cannot be distinguished, of course, from similar claims which are matched by private debts. All such claims have in the past been subject to fair-weather liquidity. Demand deposits performed all the functions of money, and claims like time deposits served as cash substitutes satisfactorily most of the time; but, during a crisis when people suspected the soundness of financial institutions and tried to withdraw deposits, the ability of all such claims to act as cash balances declined sharply. The effect was to undermine liquidity positions where claims and debts remained outstanding—over and above the shrinkage in volume of such claims (including the drain upon bank reserves and the resulting multiple contraction).

Review, Papers and Proceedings, XXXVII (May, 1947), 310.

[33] *Debts and Recovery*, pp. 14–15.

[34] As indicated above, state and local securities form a class almost by themselves; for, while these governmental units do not have the power to create money, they do have the power to tax. In any case, claims against states and local units are sometimes of dubious and fluctuating liquidity.

Nowadays, it is true, these assets are more homogeneous. Gold is unobtainable even if people prefer to hold it, deposits with banks or savings and loan associations are to a considerable extent insured, and there are some provisions for system liquidity. The sudden demotion of bank deposits to claims of very dubious liquidity is unlikely to be so severe. Nevertheless, deposits may be tied up for a short time, and the liquidity of some near-moneys may deteriorate considerably.

Since the demand deposits and near-money claims discussed in this section are matched by government debt, cash reserves, or equity investments rather than by private debt, the possibility of foul-weather illiquidity of private debt is minimized.[35] Nevertheless, even though no outstanding private debt can be called or sold, holders can sell government debt on the open market, ordinarily raising yields not only on government securities but also on corporate bonds and other private debts and arousing uncertainty about further changes in bond prices. Debtors, potential or existing, may find it more expensive or difficult to remain in debt or to sell new bonds.

Equity titles are subject to great variation in liquidity (but, of course, are not matched by any debts which can fluctuate in illiquidity). During stable or inflationary periods, some equity titles have a fair degree of moneyness. People are at times confident that they can convert them into a certain amount of cash on short notice and may allow them to substitute for cash to some extent. During periods of declining markets and uncertainty, equities possess much less moneyness, and people's cash-balance requirements increase. The quantitative significance of this factor is hard to appraise; it may easily have been underestimated in many discussions of liquidity.

Demand deposits, near-moneys, and other fixed claims matched by private debt. —In the previous section I sketched the fluctuating liquidity of all demand deposits and near-moneys. Other claims (e.g., mortgages and corporate bonds privately held, notes and accounts receivable) are also likely to have fair-weather liquidity. Short-term claims lose moneyness when collectibility seems doubtful; long-term claims lose moneyness when shiftability becomes expensive. The extreme example of loss of liquidity is complete default by the debtor.

All the claims to which this section pertains, however, are matched by private debts. To assert that loss of liquidity by these deposits, cash-surrender values, mortgages, and receivables has a net effect on liquidity positions is to assume that the offsetting obligations do not lose a corresponding amount of illiquidity. This assumption is more than conservative because, during times when the liquidity of claims comes under suspicion, the illiquidity of debt increases. When it appears that collection efforts are imminent or that debts cannot be renewed or refinanced as easily as anticipated, owing debts (whether to institutions, businesses, or individuals) necessitates acquiring higher cash balances than were previously held. Maturities can be taken lightly by debtors when money is easy, yet must be taken seriously when money is tight. Instead of a debt which can without difficulty be refinanced and which therefore requires no offsetting liquidity on the asset side of the balance sheet, the

[35] See Hart, *Money, Debt, and Economic Activity*, p. 508: "... we are in some ways safer in having bank deposits rest on government debt than we would be if they rested on private debt (bank loans). The danger that a business recession may be compounded by bank credit contraction is much reduced: our cash supply is largely deflation-proof."

debt becomes practically a subtraction from cash.

Whenever debts are voluntarily repaid, it is the debtor's liquidity desire, not the illiquidity of the obligation, which has changed. But whenever the incentive for repayment lies partly in creditors' demands and the interlocking structure of short-term debt, this pressure can be described as the "increased illiquidity of liabilities." Debtors then have to resort to distress selling of assets, distress saving of funds, and distress calling of other debts owed to them, in the general scramble to restore cash balances. The longer the maturity of the debt (unless it can be accelerated), the less debtors suffer from foul-weather illiquidity, but they cannot escape entirely. If financial institutions and others start selling long-term investments, the debtor corporations may not feel extra pressure, but yields will go up, and new financing by long-term, as well as by short-term, loans is more difficult to obtain. Increased illiquidity of debts implies reduced ability to borrow, whether for refinancing or for other purposes. Impaired liquidity positions among lenders increase cash-balance requirements for potential, as well as for existing, debtors.

C. CHANGING REAL VALUE OF CLAIMS AND REAL BURDEN OF DEBT AS PRICE LEVEL FLUCTUATES

Changing real value of claims and real burden of debts due to price-level fluctuations also affects the liquidity stock. As Pigou has urged, if the price level declines, the real value of fixed claims not matched by private debt increases, assuming that their volume does not shrink proportionately.[36] This means that the real value of the community's total stock of assets rises, with the real value of liquid assets forming a larger proportion than before. Holders of such claims should feel better off and in a more liquid position. (Aggregate demand in real terms should increase, compared to what it would have been in the absence of this effect.)

The argument does not apply to claims matched by private debts because, as the real value of these claims fluctuates, the real burden of private debts changes in an offsetting manner. If the price level falls, for instance, net worth is redistributed in favor of the creditor, but there is no change in the community's real net worth. Presumably, on balance, the community should feel neither better off nor in a different liquidity position, as far as such claims and debts are concerned. But this use of Pigou's argument assumes no money illusion. Debtors and creditors may react differently to price-level changes, especially since those who hold most of the claims are individual savers, while those who owe most of the debts are business firms. Suppose the debtors are more sensitive to changes in real burden than creditors are to changes in real value. Debtors may feel additional pressure or relief from pressure more keenly than creditors feel the increased or decreased real value of their claims. If the price level falls and this sort of money illusion exists, debtors' liquidity positions deteriorate more than creditors' liquidity positions improve, so that there is a net reduction in the community's liquidity stock, as far as claims matched by private debts are concerned.[37]

[36] "The Classical Stationary State," and "Economic Progress in a Stable Environment"; see also O. Lange, *Price Flexibility and Employment* (Bloomington, Ind.: Principia Press, 1945), pp. 13–19, and Hicks, *Value and Capital*, pp. 334–35.

[37] The existence of "money illusion" here means basing calculations to some extent upon the nominal rather than the real changes in payments, receipts,

The conclusion is that the liquidity of the community's collection of assets and liabilities is subject to change as price levels fluctuate, not only because some claims are not matched by private debts, but also because the money illusion may cause different groups to attach varying significance to fluctuations in real value and real debt burden.

Currency, demand deposits, and near-moneys not matched by private debt.[38]—The conventional Pigou argument should be applicable to all these cases. Increased real value of these holdings, in the event of a price decline, should reduce the cash-balance needs of the holders and would not be offset by a change in the real burden of any matching private obligations. The changes in liquidity position due to this effect would be in a stabilizing direction. The money-illusion argument, on the other hand, could not apply to these situations.

Demand deposits, near-moneys, and other fixed claims matched by private debt.—Changes in the real value of these claims produce no net change in the community's net worth; the Pigou argument does not apply. The money-illusion argument, however, is applicable and may explain a destabilizing or perverse influence on liquidity positions. If changes in real debt burden are felt more keenly by debtors than changes in real value are felt by holders of deposits, bonds, and receivables, the community's position may become more liquid during inflation and less liquid during deflation.

IV. NEED FOR A "NATION'S ECONOMIC BALANCE SHEET"

Problems concerned with liquidity position continually enter into policy discussions. Frequently, in the past, conclusions have been based upon scanty quantitative information. Henry Simons' picture of the "financial good-society," in which the only assets are money and equities, is invaluable as an ideal to keep before us; yet he may have exaggerated the significance of near-moneys.[39] The increased volume of near-money claims not matched by private debt does not seem to have had a marked effect on cash-balance requirements (i.e., on velocity), suggesting that people may look upon government bonds, cash-surrender values, and some kinds of time deposits predominantly as sunk investments rather than as

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or balance-sheet items—whether because of irrationality, ignorance, or expectations that some "normal" price level will return. If the price level falls, creditors may feel neither better off nor more liquid because they believe prices will return to "normal," but debtors may have no option, since repaying interest and principal out of reduced incomes renders them palpably worse off and less liquid (cf. James Tobin, "Money Wage Rates and Employment," in *The New Economics*, ed. Seymour Harris [New York: Alfred A. Knopf, Inc., 1948], p. 581).

[38] Equity titles do not enter this discussion. Price fluctuations should produce no change in real value of equities, on the average, and there is no offsetting debt whose real burden might change. Equity titles are the articles whose fluctuating prices make up a changing price level. Although uniform, proportionate changes in all prices never occur, there is no presumption that properties in the aggregate acquire or lose real value during fluctuations. (Use of an actual price index as a deflator would show changes; for one thing, even if prices of all properties entered into the index, weights could hardly reflect changes in the composition of equity titles.) In any case the original argument regarding fixed claims assumed that increased real value of claims without any altered real value of equities increased the proportion of asset values held in comparatively liquid form.

[39] See Simons, "Debt Policy and Banking Policy," *Review of Economic Statistics*, XXVIII (May, 1946), 85–86. Apparently his emphasis on near-moneys was, in large part, the reason he referred to private banks and insurance companies as ". . . the two forms of private business which are least essential to, if not incompatible with, a free-enterprise, free-market system," in his article, "The Beveridge Program: An Unsympathetic Interpretation," *Journal of Political Economy*, LIII (September, 1945), 217.

liquid reserves.[40] Foul-weather illiquidity on the liability side, also stressed by Simons, may be more significant than the fluctuating volume of checking accounts and near-moneys on the asset side.

Moreover, in the past, conclusions have often been based upon partial analysis of liquidity positions. We have treated similarly types of claims and debts which were not, in fact, similar and have neglected or underemphasized the changes in certain balance-sheet items. I believe the discussions of postwar banking policy may be an example of this. The federal government used part of its surplus to retire bonds held by member banks, restoring their reserves so that they were able to expand loans to private borrowers. As the surplus drained off demand deposits, the banking system poured them back into circulation. Since the stock of adjusted demand deposits plus currency in circulation did not decline, the net effect did not appear to alter the economy's liquidity position:

> In short, there is no direct deflationary effect from using a government surplus to repay short-term securities held by banks. There is only an opportunity for the Federal Reserve to mop up some bank reserves without forcing the banks to call loans or sell securities in the open market. There is, however, a by-product in the reduction of equities and increase of debt to the banking system.[41]

Dr. Hardy's main point was sufficiently important to overshadow his last sentence. Deflationary effects of a Treasury surplus on liquidity positions have since been associated almost exclusively with retiring debt held by the Federal Reserve banks or building up Treasury deposits with the Reserve banks.[42] My analysis suggests that we should not entirely overlook the deflationary influence of increased private indebtedness to the banking system. The original Phillips analysis implies that these debts probably require higher cash balances on the part of borrowers than would be required in the absence of the debts. Retirement of bonds held by member banks out of tax receipts is inflationary compared to paying off debt held by the Federal Reserve banks; yet the transaction as a whole is deflationary. Similar reasoning puts retirement of bonds held by insurance companies, savings and loan associations, or savings banks in a slightly different light. It is true that "this process of shifting out of Government securities results in the creation of bank deposits and bank reserves and is just as inflationary as bank credit expansion,"[43] but again the resulting increase of private debt (when the institutions lend the funds obtained) must increase the cash-balance requirements of the borrowers.

[40] Cf. A. G. Hart, "Postwar Effects To Be Expected from Wartime Liquid Accumulations," p. 344.

As life insurance companies and mutual savings banks increased their holdings of government bonds (near-moneys), they showed no consistent tendency to lower the proportion of assets held in the form of cash, though these institutions hoarded and dishoarded cyclically, much as others tried to do (Jones, "The Optimum Rate of Investment, the Savings Institutions, and the Banks," pp. 325–29). Warburton urges that savings bonds, on account of their yield, do not function as important near-moneys ("Quantity and Frequency of Use of Money in the United States, 1919–45," *Journal of Political Economy*, LIV [October, 1946], 435–37).

[41] C. O. Hardy, "Fiscal Operations as Instruments of Economic Stabilization," *American Economic Review, Papers and Proceedings*, XXXVIII (May, 1948), 399.

[42] See the repeated references to using the surplus to retire bonds held by the Federal Reserve banks in the article, "Bank Credit Developments," *Federal Reserve Bulletin* (October, 1948), pp. 1205–16. Everyone seems anxious to avoid any suggestion of the errors of the 1945 *Annual Report* which Hardy pointed out.

[43] *Ibid.*, p. 1216.

A shift from liquid assets offset by public debt to similar assets matched by private debt puts the economy in a less liquid position.[44]

Purchase of government securities by the Federal Reserve banks, though clearly putting the banking system and the public in a more liquid position, makes possible a rise in private debts whose opposite impact on liquidity positions should not be overlooked. In view, too, of the potential illiquidity of private debt, it seems rash to assert confidently that ". . . stability in the Government bond market . . . contributes to the underlying strength of the financial structure of the country."[45] It should at least be acknowledged that the growth of private indebtedness facilitated by supporting government bonds contributes to the underlying weakness of our liquidity position.

To minimize these deficiencies in the future—i.e., to compile relevant quantitative information systematically, to focus attention on liquidity position as a whole, and to prevent excessive emphasis on an oversimplified income-expenditure analysis—I suggest that a "nation's economic balance sheet"[46] would be an appropriate companion statement to the "nation's economic budget."

In drawing up and interpreting such a financial statement, it might be well to follow business practices as closely as possible.[47] We are trying to isolate balance-sheet relationships which influence business and individual actions. Perhaps we should classify and interpret the aggregates in much the same way that a businessman classifies and interprets his balance-sheet items.[48] (As a working hy-

[44] Several other factors, until 1948–49, had been reducing our liquidity: (1) the decline in real value of currency and claims not matched by private debts as prices rose; (2) reduced liquidity of banks as portfolios included fewer governments, as interest rates on short-term governments were permitted to rise, and as reserve requirements were increased (until recently); (3) reduced liquidity of long-term corporate bonds, common stocks, and other equities as yields rose and uncertainty grew; (4) reduced liquidity of inventories; (5) increased illiquidity of short-term debts (implying higher interest charges and decreased "availability" of funds in other ways).

[45] *The Economic Report of the President to the Congress, January 7, 1949* (Washington: U.S. Government Printing Office, 1949), p. 11.

[46] Interesting developments in this direction include *The Balance Sheet of Agriculture* (Miscellaneous Pub. No. 672 for 1948, by the Bureau of Agricultural Economics, U.S. Department of Agriculture); and the statements of "Current Assets, Current Liabilities, and Working Capital of American Corporations," quarterly releases by the Securities and Exchange Commission.

Townsend-Skinner's Financial Accounting is an attempt to use part of the economy's balance sheet in diagnosing stock-market fluctuations (see Garfield A. Drew, *New Methods for Profit in the Stock Market* [Boston: Metcalf Press, 1948], pp. 15–16).

[47] F. A. Lutz preferred to use ratios of cash to payments (or cash to sales) to discover whether corporations had in the past possessed "free" cash and wrote that quick or other asset-debt ratios ". . . are definitely of little importance to our special problem. Business management also gives little consideration to such ratios, except as it is necessary to do so to meet requirements of credit grantors" (*op. cit.*, p. 63).

Ratios of liquid assets to past payments, however, are backward-looking; and ratios of liquid assets to future payments, though subsequently calculable for examining past behavior, are unavailable for appraising current liquidity position. Surely current liabilities constitute a useful estimate of expected payments—that is, of those already scheduled.

Cf. H. W. Arndt, "The Concept of Liquidity in International Monetary Theory," *Review of Economic Studies*, XV (1947–48), 26: "It is usual to define the 'liquidity position' of a bank, not in terms of the disposition of its assets only, but in terms of certain *ratios* of assets to liabilities, especially the ratios of cash to deposits and of liquid assets (or, conversely, advances) to deposits. It is clear that this is also the only feasible approach to the concept of 'liquidity position' in the international context. . . . In practice, if the concept is to be of use, it will be necessary, internationally as much as in domestic theory, to devise some rough and ready measure of liquidity position, in the form of some significant ratio."

[48] For example, there could be, on the asset side, currency in circulation (outside the Treasury and

pothesis, we might assume that individuals and small businesses follow, though less consciously and less religiously, the same principles supposedly followed by corporation comptrollers and executives.) However, for such a combined national balance sheet it would be advisable to have, wherever possible, some additional breakdown according to owners in the case of assets and according to owers in the case of liabilities. Ideally, each group of claims (debts) would be further subdivided into holdings by (obligations of) financial institutions, businesses, farmers, and other individuals.

It is especially important that the claims held, and debts owed, by financial institutions be segregated—probably by having two separate balance sheets, the nation's financial institutions balance sheet and the nation's economic balance sheet, the latter showing only assets and liabilities on the books of nonfinancial businesses and individuals. Ratios computed from the two segregated balance sheets should prove more informative than ratios computed from a combined balance sheet.

In interpreting his balance sheet, the businessman uses various ratios, not just one ratio, to help gauge his liquidity position. He does not, of course, blind himself to other guides—e.g., changes in individual items, price levels, liquidity of receivables, and illiquidity of payables. His allowance for the last two factors, which do not appear on the balance sheet, shows up through his requiring higher asset-debt ratios during deflation than during inflation. During periods of contraction, alert businessmen, bankers, and investors, recognizing impaired liquidity positions due to altered collectibility of claims and renewability of debts (as well as having unfavorable anticipations), probably require higher asset-debt ratios of themselves and of those to whom they extend credit. Again the economist can follow suit. Within the nation's economic balance sheet, several ratios can be observed—not mechanically but in the light of changes in individual items, price levels, and the moneyness of assets and liabilities.

To recapitulate, this analysis has indicated that changes in liquidity may come about through changes (1) in the stock of assets (of varying degrees of liquidity) diluted in some cases by changes in the volume of liabilities (of varying degrees of illiquidity); (2) in the liquidity of various assets which remain outstanding, reinforced in some cases by simultaneous changes in the illiquidity of matching debts; (3) in the real value of claims not matched by private debts and, through the money illusion, in the real value and debt burden of matching claims and debts. I believed it useful to hold other things constant while examining variations in particular balance-sheet items, but I pointed out that other things do not usually remain the same. Changes in one group of assets or debts may be ac-

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Federal Reserve banks); demand deposits; near-money claims (maturing in one year or less from the date of the balance sheet) against the government, against financial institutions, and against nonfinancial businesses and individuals; business inventories; long-term claims (maturing more than one year hence) against the government, against financial institutions, and against nonfinancial businesses and individuals; equity shares listed on major stock exchanges; other equity titles. On the liability side, one could list short-term debts (maturity date one year or less hence) to commercial banks, other financial institutions, businesses, and others; and long-term debts (maturity date more than one year hence) to the same creditors. For an example of the possibilities in this direction see Elwyn T. Bonnell, "Public and Private Debt in 1947," *Survey of Current Business*, XXXVIII (October, 1948), pp. 20–23, and the National Bureau's projected preparation of a National Balance Sheet as of the end of selected years (*Agricultural Finance Review* [November, 1947], pp. 93–94).

companied by offsetting or reinforcing changes in some other group of assets or debts.

Thus the variety of possible influences and the complexity of their interrelationships suggest that no single index (such as the quantity of money, the real value of the stock of money plus government bonds, or the ratio of aggregate liquid assets to aggregate private debt) can tell us as much as we need to know about liquidity position. A study of the national balance sheet as a whole, using several ratios or indexes, along with national income and expenditures, is more likely to throw light on fluctuations. Much discussion has centered around the nation's economic budget, comparable to a corporation's income or sources-and-applications-of-funds statement. A nation's economic balance sheet might be a wholesome supplementary exhibit.

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MATHEMATICAL ECONOMICS BEFORE COURNOT

ROSS M. ROBERTSON[1]

I

IT IS generally held that no important mathematical analysis appeared in economic literature before Cournot published his *Recherches* in 1838. Although the earlier writers were undoubtedly less important than Cournot, their contributions ought not to be neglected by anyone interested in the early development of economics.

The mathematical writers of the period before 1838 are listed, with a few exceptions, in the Jevons-Fisher bibliography of mathematico-economic literature.[2] The strong implication of this specialized bibliography—that the writings which are not included contain little of interest—has proved to be correct. There is nothing in the literature of political arithmetic, mercantilism, physiocracy, or the English classical school which may properly be regarded as mathematical.

To be sure, there were often slight indications of a mathematical bent or beginning, but no serious attempt was made to apply a mathematical method. The published works of writers on political arithmetic contributed nothing, contrary to what we might expect in view of their preoccupation with quantities. In the literature of the mercantilists there is frequently a consciousness of relationships between varying quantities which is promising, but mathematical use of the relationships is lacking. The physiocrats, too, are disappointing. The only one mentioned by either Jevons or Fisher is Condorcet, a lesser figure on the periphery of the main group. The brief, but complicated and obscure, passage in Condorcet's *Vie de Turgot* which outlines the French statesman's tax theory would lead one to expect an analysis in Turgot's works similar to that of Condorcet. One finds little in Turgot, however, of an explicitly mathematical nature; in only one unimportant instance does he use algebra in economic analysis. Quesnay and his associates were on the threshold of the mathematical method in their consideration of economic aggregates; nevertheless, they used only arithmetic devices, and, although the *Tableau économique* could well be translated into algebra, the physiocrats did not do this. Professor Knight has so well pointed out the mathematical deficiencies of the classical economists that it appears unnecessary to comment further on them.[3]

II

Generally speaking, pre-1838 attempts to use a mathematical method were made

[1] I am indebted to Professor R. S. Howey, of the University of Kansas, for criticisms and suggestions in the preparation of this article.

[2] For the bibliographies see William Stanley Jevons, *The Theory of Political Economy* (4th ed.; London: Macmillan & Co., Ltd., 1931), pp. 322–26, and A.-A. Cournot, *Researches into the Mathematical Principles of the Theory of Wealth* (New York: Macmillan Co., 1897), pp. 173–76; see also E. R. A. Seligman, "On Some Neglected British Economists," *Economic Journal*, XIII (September, 1903), 352, for mention of Samuel Gale and the anonymous English writer, "E. R."

[3] Frank H. Knight, "The Ricardian Theory of Production and Distribution," *Canadian Journal of Economics and Political Science*, I (February and May, 1935), 3–25 and 171–96.

by those who adhered to no "school," who, on the whole, took up economic investigation with few prejudgments, and who frequently achieved amazing originality. Of the few other generalizations that can be made on their background, the clearest one is that nearly all were university professors or engineers. In their economic analysis they have two characteristics in common: first, they

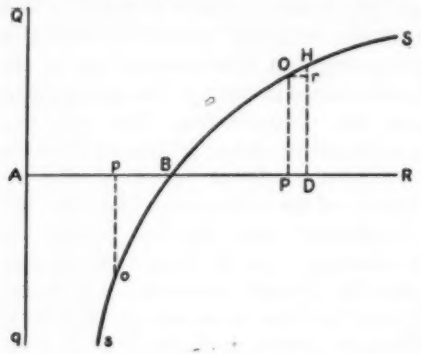

FIG. 1

use mathematical notation, and, second, each author shows a consciousness of the necessity of considering an economic problem in terms of the relevant dependent and independent variables, putting them into functional relationship by means of some principle thought to be applicable. Although the relationships so expressed are usually in a crude form, *conscious functional thinking* characterizes the work of all the early mathematical writers.

It would be misleading, however, to give the impression that their contributions to economic methodology were confined to the general ones given above. They made three innovations, later important in economics, which are ordinarily attributed to Cournot and which, with one exception, have gone almost unnoticed to the present day. These innovations were the employment of the calculus, the writing of equations without insisting that functions be defined by analytic formulas, and the use of geometry to express two-variable functions.

III

The only early use of the calculus in economics generally known is that of the Swiss mathematician, Daniel Bernoulli, whose analysis of utility in 1738 resulted from a consideration of games of chance rather than of economic problems. The central idea, well known largely because of Marshall's reference, is that, in considering additions to total satisfaction resulting from incremental additions to one's wealth, it is important to include as a criterion of measurement the amount of one's wealth as well as the size of the increment. The following account is taken from the translation of Bernoulli's *Specimen theoriae novae de mensura sortis* (1738) into German from the Latin.[4]

In Figure 1 let AB represent the wealth which an individual possesses *before* engaging in an even bet. Construct the curve BS so that the ordinates represent the "advantages" which result from gains in wealth. The question now is: What *ought* to be the sum which should be gambled if the amount that he has a 50 per cent chance of winning is BP? To discover this sum, continue the curve BS in the opposite direction in such a way that the abscissae represent the amounts paid for the privilege of playing the game and the ordinates the "disadvantages" resulting from such payments. Since in an even bet the "disadvantage" of the payment must equal the "advan-

[4] *Die Grundlage der modernen Wertlehre:* Daniel Bernoulli, *Versuch einer neuen Theorie der Wertbestimmung von Glücksfällen* (*Specimen theoriae novae de mensura sortis*), translated from the Latin by Dr. Alfred Pringsheim (Leipzig: Duncker & Humblot, 1896), pp. 31–37.

tage" of the possible gain, $po = PO$. Therefore, Bp is the largest amount that should be paid for a 50 per cent chance of winning BP. Thus Bernoulli concludes that a prospective wager may not be considered simply on its own merits and that it is wrong for a person to pay a sum precisely equal to the sum to be gained, *except* in the case in which the curve sBS is a straight line or in which the sum wagered is very small relative to income. Since Bernoulli was writing to solve the St. Petersburg Paradox, it was necessary for him to show not only that the curve sBS was not a straight line but that it was concave from above, that is, that the marginal utility of income diminishes as income increases.[5]

From this point on, the analysis is in terms of infinitely small quantities; BP and BD are considered as almost equal, so that their difference, PD, becomes as small as desired. And here, consequently, is where the first employment of the calculus was made in anything that resembled an economic problem. If Or is drawn parallel to BR, rH becomes the small "advantage" which accrues to anyone with a "fortune," AP, upon the acquisition of a small gain, PD. Bernoulli supposes that the "advantage" is directly proportional to the gain, PD, and inversely proportional to the "fortune," AP. An assumption of Bernoulli's, then, determines the shape of this curve. It is, of course, this assumption of diminishing marginal significance of income that gave him a solution to the St. Petersburg Paradox.

Letting x be the amount of wealth and y the "advantage" or "disadvantage," the supposition that any additional "advantage" from an addition to wealth is directly proportional to the increment of wealth and inversely proportional to the amount of wealth can be written, where the addition approaches zero, as

$$dy = \frac{b \cdot dx}{x}.$$

Integrating, the equation for the curve is

$$y = b \cdot \log \frac{x}{a},$$

which is a logarithmic curve. Since it is not a straight line, a bet cannot be considered on its own merits as fair. This, then, was the first use of calculus in a problem which, if it were not economics at the time, at least subsequently turned out to be.

There was a second employment of the calculus in economics in the eighteenth century which, while not presented in altogether satisfactory detail, must be mentioned. The passages of interest are

[5] Had he considered the prevalence of both gambling and insurance by the same person, he might have concluded, as did Milton Friedman and L. J. Savage, that for some amounts of income the curve was convex and for others concave (see their "Utility Analysis of Choices Involving Risk," *Journal of Political Economy*, LVI [August, 1948], 279–308). These authors also point out that the essential contribution of Bernoulli was not, as Marshall apparently thought, the statement of the law of diminishing utility (the shape of the curve) but that it lay rather in showing the importance of the relation of utility to income. As Bernoulli noted and as Friedman and Savage (and, for that matter, Marshall) point out, Bernoulli was anticipated in his essentially valid contention by Gabriel Cramer.

The first crude expression of this basic idea appeared, so far as I can tell, in a paper by Edmé Mariotte entitled *Essai de logique*, published at Paris in 1678 (see *Œuvres de M. Mariotte* [Leide, 1717], pp. 666-67). Professor Jacob Viner has called my attention to the elaborate consideration of the same problem by the distinguished French naturalist, Georges Louis Leclerc Buffon, in his *Essai d'arithmétique morale*, published in 1777. Although familiar with the principles of the calculus, Buffon did not use this tool in his analysis. His conclusion is essentially that of Bernoulli, but his treatment of the diminishing marginal importance of money is much more detailed and will repay careful reading (see *Œuvres complètes de Buffon*, XII [Paris: Garnier Frères, Libraires], 154–78).

found in some editorial notes, written by the Italian mathematician, Paolo Frisi, which appear as mathematical commentaries on certain sections in Pietro Verri's *Meditazioni*.[6] Verri had developed the proposition that the price of a good is directly proportional to the number of buyers of the good and inversely proportional to the number of sellers. He further contended that the ends of the state could best be served by bringing about a situation in which the number of sellers of each good was in the greatest possible proportion to the number of buyers, the most obvious benefit of the resulting low prices being the stimulus which would be afforded to the export trades. There are limits, of course, to the increase which can take place in the number of sellers of a given good, and there are ethical objections to reducing by legislative action the number of buyers of a particular commodity.

Verri used no mathematical notation, but Frisi stated his proposition with regard to price determination as a formula, $P = C/V$, where P is the price, C the number of buyers (*compratori*), and V the number of sellers (*venditori*). Frisi first attempts to show that the expression $P = C/V$ is at least approximately true. In most of his annotations thereafter he is concerned with maximization and minimization of economic quantities. Since P can in no significant sense be negative, the minimum value of the function will be zero.[7] But there is a "practical" minimum greater than zero, for it is absurd to think of prices being reduced to nothing by state action. Frisi is no more explicit than Verri when it comes to giving rules for achieving a proper minimum. He simply says that, *once it is achieved*, the state can *maintain* the proper minimum by properly offsetting changes in the number of sellers (or buyers) by proportional changes in the number of buyers (or sellers). And here is where he first uses an idea from the calculus. Considering the differentials, dP, dC, and dV, we have, in the case of a minimum,

$$dP = \frac{V\,dC - C\,dV}{V^2} = 0,$$

and therefore

$$\frac{dC}{dV} = \frac{C}{V}.$$ [8]

These relationships would hold, no matter what the value of P, but it is at the minimum value that they are of interest.

Toward the end of his book Verri stated that the problem of political economy is to increase production as much as possible with a minimum expenditure of labor. Frisi commented that this proposition, reduced to analytical terms, would be of the class then referred to as an "isoperimetric problem." If R is the quantity of production and T the quantity of labor and if A and B are constant, then in the case that R is a maximum and T a minimum ". . . we would know the differential equations of the problem $dT = 0$, $dR = 0$, and the integral equation $R + AT = B$."[9] It can

[6] Pietro Verri, *Meditazioni sulla economia politica* (6th ed.; Livorno, 1772), pp. 37–39, 43, 84, 92, 97–98, 134, 152, 232–33, and 237–53.

[7] Strictly, the function $P = C/V$ has no maximum or minimum if C and V are unrestricted. Frisi pays no attention to the nature of possible restrictions. In a paper given before a learned society in 1796, the mathematician Pietro Ferroni mentions the mathematical difficulties involved in treating maxima and minima in such a case (see Pietro Ferroni, "Esame di alcuni passi della Meditazioni sulla economia politica del conte Pietro Verri," *Scrittori classici italiani di economia politica*, parte moderna, XVII (Milan, 1804), 391–92.

[8] Verri, *op. cit.*, p. 43.

[9] *Ibid.*, p. 134. When Frisi imposes the condition $R + AT = B$, he obviously does not consider the possibility of diminishing returns. In a later consideration of the application of labor to agricultural

only be assumed that Frisi considered R and T as functionally related to other variables. He did not discuss such relationships, however, and was content to state a broad principle without embellishment.

The third writer in point of time to use the calculus was Georg von Buquoy. Most of von Buquoy's works are on what he calls the "technical" part of political economy. The word "technical" is an apt one, for he discusses the laws of physics applicable to the proper loading and towing of wagons, the rules for pressing oil and wine, for running sawmills, for reducing iron ore, etc. Even his brief section on commerce deals largely with such things as measures, weights, tariff rate-making, and the like. In his section on agricultural techniques, however, there is a brief passage in which he treats the problem of maximization. This analysis is so strikingly modern in all its implications that it deserves a detailed report.[10]

The author wishes to put depth of plowing on a "rational" basis so as to determine what depth may be of greatest advantage to the farmer. In order to do this, one must know according to what "law" yield increases as depth of plowing becomes greater and also according to what "law" costs increase with increasing depth. With regard to total revenue, von Buquoy postulates that, as depth of plowing increases, total revenue increases but in "ever decreasing proportion" until beyond a certain depth no further gain in revenue is forthcoming. With regard to total cost, as depth of plowing increases, total costs increase in "ever increasing proportion," so that at a relatively great depth the cost is very high. Then Buquoy reasons as follows:

> Let the law according to which the value of the crop depends upon the depth of plowing be given by the equation $y = f(x)$, and let the law according to which costs depend upon depth of plowing be expressed by the equation $Y = F(x)$. Then the net revenue can be generally expressed at any depth by $f(x) - F(x)$, from which that value can be found at which the net revenue is a maximum. And this maximum will be at that value of x at which the first derived function of $f(x) - F(x)$ disappears and at which simultaneously the second derived function becomes negative.[11]

Except for the fact that he is not considering total revenue and total cost as a function of output but rather as a function of depth of plowing (i.e., of a technical operation), this is the marginal-cost-equals-marginal-revenue statement of the problem of maximization, with both the necessary and sufficient conditions given.

The earliest English writer to use the calculus in economics was T. Perronet Thompson.[12] The importance of the article lies in Thompson's enunciation of the idea that, if total gain is to be maximized, marginal revenue and marginal cost must be equated. Like Buquoy, Thompson expressed the necessary condition for a maximum difference between total gain and total cost.

"On the Instrument of Exchange" was written ostensibly as a comment on two papers which had appeared earlier—one by Sir John Sinclair, the other by William Huskisson.[13] Sinclair had advanced

land (p. 156) he lays down precisely the same condition.

[10] It will be noted that von Buquoy employs the general concept of a function in this passage. A fuller treatment of his use of this device will be given presently.

[11] Georg von Buquoy, *Die Theorie der Nationalwirtschaft* (Leipzig, 1815), p. 54.

[12] "On the Instrument of Exchange," *Westminster Review*, I (April, 1824), 171–205.

[13] *Ibid.*, pp. 171–72. The articles referred to were: Sir John Sinclair, "On the Means of Arresting the Progress of National Calamity," *Pamphleteer*, X (1817), 419–41; and William Huskisson, *The Ques-*

the view that the quantity of money in the English economy should be increased or the British people would suffer dire economic consequences. Huskisson had taken the opposing view that the currency of a country could not be depreciated without harmful results to all classes within the state. Thompson's objection to both papers is that neither author states in a systematic way the effects which alterations in the quantity of money may have on an economy.

Thompson suggested that the state might issue paper money for its current expenses and agree to receive it for taxes. Since some of the paper money would remain in circulation, the state would gain. The people might not like this, but, after a short period of uneasiness and agitation, the notes would continue to serve as the instrument of exchange and would circulate at par so long as no greater quantity was issued than was received for taxes in any given period of time. Thompson said that it was as well established as any economic fact could be that an irredeemable paper currency would circulate without depreciation so long as it was made legal tender and so long as the quantity was not increased appreciably.

However, if the government were to issue in any given period of time a greater amount of paper money than was received in that period in discharge of taxes, an increase of prices would result. If four thousand units were in circulation where three thousand circulating at par would be sufficient—then the money would be depreciated by one-fourth, or, what is the same thing, the prices of commodities would rise by one-third.[14]

Thompson then examines in detail the consequences of a systematic increase of an irredeemable paper currency. Suppose that the entire monetary medium consists of paper money circulating at par. Suppose, further, that the government continues to issue—say, daily—a number of notes in excess of the number returned in discharge of taxes:

Let p equal the legitimate issue—i.e., the number authorized by the return taxes.

Let s equal the daily superfluous issue—i.e., the number over and above the legitimate issue.

Let t equal the number of days in which there has been a superfluous issue.

Let z equal the fraction which expresses the depreciation.

Now the purchasing power of the superfluous issue during any small period of time, dt, is $sdt - (sz)dt$. The purchasing power lost on the remainder of the issue is equal to $(pz)dt$. The gain (dG) during any small period of time, dt, is

$$dG = s \cdot dt - (s + p)\, dt \cdot z\,.$$

When this expression is integrated, the gain, G, can be expressed in terms of the other variables. The problem, however, is not to measure the gain but to determine at what point the government should stop the superfluous issue in order to maximize the gain. This is an important use of the calculus in economics today.

Thompson states that "when the depreciation is such that $(s + p)zdt = sdt$, the daily gain will be at an end." This is the condition that $dG = 0$. Here, of course, we have a situation different from that in which the firm seeks, in the short period, an output at which it maximizes net revenue; for in this instance it is the government which seeks to maximize a gain by fiscal manipulation through time. The principle involved, however, is pre-

tion concerning the Depreciation of Our Currency Stated and Examined (new ed.; London: J. Murray, 1819).

[14] *Ibid.*, p. 185.

cisely the same; it is the necessary condition for maximum G.

IV

The second considerable addition to economic methodology consists in the use of the general concept of a function. The importance in analysis of equations not defined by analytic formulas is apparent from the inaccuracies and even absurdities of those authors who, writing their equations in less general form, were led to state their "laws" in terms of strict proportion. In this connection Nicholas-François Canard, Francesco Fuoco, Pietro Verri, and Major General Henry Lloyd come to mind. An obscure German, Joseph Lang, in his *Grundlinien der politischen Arithmetik* (1811) used nonanalytic functions in a study of aggregate quantities, employing a single letter, instead of the modern notation, to represent a varying portion of a particular quantity. But by far the most thoroughgoing explanation of the possibilities in using a general notation is found in von Buquoy's work. He gives detailed instructions for the use of this device; for he says that is is often sufficient in the study of political economy to use ". . . algebraic formulas as sentences expressed in symbols."[15] For this purpose he tells us:

> Let $f(X_+)$ signify a quantity dependent upon X in such a way that, as X increases, the quantity also increases.[15a] For example, we can understand by $f(X_+)$ the market price of a commodity and by X the quantity of an actual demand. On the other hand, $f(X_-)$ indicates a quantity which depends upon X in such a way that, as X increases, the quantity decreases. For instance, the market price of a commodity decreases if the competition, X, of sellers increases. The symbol $f(X_{+a-})$ means that a certain quantity depends on X in such a way that, as X increases, it increases to a certain limit a; beyond this limit, however, the quantity decreases as X increases. The symbol $f(X_{-b+})$, on the other hand, indicates a quantity which decreases to a certain limit b as X increases; beyond this limit it increases as X increases. Thus as division of labor increases, natural price is lowered, but a limit will be reached beyond which natural price again increases.
>
> The application of such symbolic expressions, which conveniently express a whole chain of ideas in a single expression, becomes a great aid in the presentation of abstract combinations. . . . Thus, for instance, $F[f_+(X_+)]$ indicates a quantity which increases as $f(X)$ increases, where X itself is an increasing quantity. . . . Now this can be put a little more clearly. Instead of saying $Z = F[f_+(X_+)]$, have $Z = F(y_+)$ and $y = F(X_+)$. Similarly, instead of writing $U = F[f_+(X_-)]$, one could write $U = F(y_+)$ and $y = f(X_-)$; from which it would follow that, with an increasing value of X, the value of $f(X)$ decreases, and therefore U decreases. Here, for instance, one could express by X the competition of sellers, by $f(X)$ the market price of a good, and by U the business profit of the sellers. If one writes $Y = \phi(y_+, x_-)$ he expresses the fact that, as y increases, Y increases and, as x increases, Y decreases. Thus one might express the relationship between Y, the market price, and the competition among buyers and among sellers, where y stands for the former and x for the latter. Finally, should these quantities be substituted in the expression $U = F(Y)$, there would result $U = F_+[\phi(y_+, x_-)]$; that is, with competition y among buyers and x among sellers, market price at the same time grows and diminishes, and so consequently does the business gain of the individual merchant.[16]

Thus we find for the first time instructions for the use of the general concept of

[15] Georg von Buquoy, *Erläuterung einiger eigenen Ansichten aus der Theorie der Nationalwirtschaft, Zweyter Nachtrag zur Theorie der Nationalwirtschaft* (Leipzig, 1817), p. 335. Buquoy remarks that it is no wonder that J.-B. Say became angered at the misuse of mathematics, for he had not seen it properly employed in the statement of the great general laws having to do with economics.

[15a] [To avoid casting special type, certain changes have been made in Buquoy's notation.—EDITOR.]

[16] *Ibid.*, pp. 335–36. This is a fairly free translation, but it follows the text closely enough to be considered a direct quotation.

a function in economic theory. It is worthy of note, however, that von Buquoy does not employ this device in his principal section on "natural price." In discussing "natural price," he states algebraically the "costs" at different stages of production, where costs include rent, profit, and interest, as well as current outlay on labor and raw materials. Natural price is equal to a summation of all the money costs encountered at various stages of production.

V

Aside from the geometric treatment of Bernoulli given above, only two other uses of geometry have been found in the mathematical literature before Cournot. One of these is in a postscript to Thompson's article "On the Instrument of Exchange," which appeared in Volume XII (1830) of the *Westminster Review*. It is used in a simple exposition of an idea regarding the taxation of income.

The third use of geometry is suggested in an interesting article of Cesare Beccaria, an Italian economist and sociologist of the eighteenth century. Becarria's brief mathematical work is his "Tentativo analitico su i contrabbandi," published in the periodical, *Il Caffè*.[17] This passage is of special interest because it is among the first attempts to analyze a specific problem in public affairs with the tools of pure theory.[18]

As an example of the way in which an economic problem may be solved analytically, Beccaria investigates the profitability of smuggling. When the state imposes a duty on goods, it ordinarily makes evasion punishable by the loss of the merchandise involved. The author proceeds to consider what portion of a given commodity a smuggler can afford to lose by confiscation and still be as well off as if he had brought the goods in legitimately and had paid the tax:

> Let u equal the cost of all the goods which the merchant will attempt to smuggle into the country.
>
> Let t equal the total amount of tax if all the goods came through customs.
>
> Let x equal the cost of the merchandise required to be smuggled if the merchant is to break even.
>
> Finally, let d equal the difference between the tax and the cost of the merchandise.

Now "we shall have for the condition of the problem the equation $x + tx/u = u$."[19]

Multiplying by u, we have

$$ux + tx = u^2,$$

$$x(u + t) = u^2,$$

$$x = \frac{u^2}{u + t}.$$

Now the tax could be equal to the value of the good, that is, $t = u$. But the tax might be higher than the value of the good by the quantity d, i.e., $t = u + d$;

[17] Cesare Beccaria, "Tentativo analitico su i contrabbandi," *Il Caffè*, I (Brescia, 1765), 118–19. This article may also be found in *Opere di Cesare Beccaria* (Milan, 1822), I, 428, and in *Scrittori classici italiani di economia politica*, parte moderna, XII (Milan, 1804), 235.

[18] Other aspects of the important historical problem of smuggling are investigated algebraically by Guglielmo Silio in his *Saggio sull'influenza dell'analisi nelle scienze politiche ed economiche*, which was published in 1792 at Palermo. I have not discussed the Silio treatment, since, upon Silio's admission, it was suggested by Beccaria's analysis and is similar to it.

[19] Cesare Beccaria, *Opere di Cesare Beccaria*, I, 429. The condition to be fulfilled is that the merchant shall break even. The amount of his loss is the cost of the goods to be imported minus the cost of those he must successfully smuggle in order to lose nothing, or, in symbols, $u - x$. His gain equals the fraction of the goods that are not seized by the customs officials multiplied by the total tax he would have paid had all goods entered legally, or, in symbols, tx/u. When his loss equals his gain, $u - x = tx/u$. Hence the expression $u = x + tx/u$ is the condition that the merchant make neither profit nor loss on his venture.

or it might be lower than the value of the good, i.e., $t = u - d$.

We may substitute three values of t in the equation

$$x = \frac{u^2}{u+t}.$$

When $t = u$, then

$$x = \frac{u^2}{u+u} = \frac{u^2}{2u} = \frac{u}{2}.$$

When $t = u + d$, then

$$x = \frac{u^2}{u+u+d} = \frac{u^2}{2u+d},$$

or less than $u/2$. When $t = u-d$, then

$$x = \frac{u^2}{u+u-d} = \frac{u^2}{2u-d},$$

or more than $u/2$.

Those who wish, says Beccaria, may plot the equation $ux + tx = u^2$. He gives full instructions but does not draw the graph.[20] Guglielmo Silio, commenting on the fact that Beccaria described the curve but neglected to draw it, actually plotted the equation. His figure is reproduced exactly as Figure 2.[21] It will be seen that $AB = u$, that BC and BF are values of t, and that CD and FG are values of x. Thus $(AB + BC)CD = AB^2$ or $(u + t)x = u^2$. When t has very large values, x becomes very small; when $t = 0$, $x = u$.[22] Such information would be of interest to the tariff-maker, according to both Beccaria and Silio, because he would have the advantage of knowing how much he should have to fear from smugglers under various tariff rates.

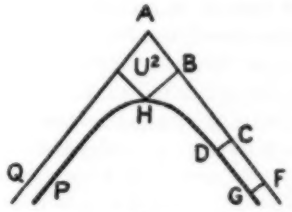

FIG. 2

VI

Thus we have before Cournot a clear use of three important tools of modern economic analysis. In view of the present ubiquity of geometric presentation in economic theory, it is worthy of comment that the early writers thought more in terms of analytical (algebraic) method than they did in terms of the geometric. There were, however, some attempts to picture functional relationships by the use of geometry.

Little interest in general equilibrium analysis was shown by the early writers. Broadly speaking, the early mathematical economists were concerned with the relationships among a small number of variables. They were preoccupied, for the most part, with getting answers. Their comprehension of the essential unity of an economic system had not reached the point of considering one variable of the system as being functionally related to all the other variables in that system. In 1781 a French engineer, Achille-Nicolas Isnard, came as close to the general equilibrium as anyone.

[20] *Ibid.* "Supponendo nell'equazione $ux + tx = u^2$ indeterminata la t e la x e costante la u; il luogo dell'equazione sarà ad una iperbola fra gli assintoti, di cui le abscisse t prese sull'assintoto da una distanze u dall'angolo assintato, più la medesima distanza, saranno alle ordinate x parallele all'altro assintoto in ragione costante, cioè come il quadrato della potenza u. L'ispezione della figura, in chi la voglia costruire, rischiarirà tutti i differenti case dell'equazione."

[21] Silio, *op. cit.*, p. 151. The librarian of the University of Bologna was good enough to make a photostatic copy of this essay, and this copy is in the University of Tennessee library.

[22] *Ibid.* The curve of the equation $ux + tx = u^2$ is a rectangular hyperbola, as may be readily verified. The Silio figure was constructed inaccurately, for the asymptotes AF and AQ do not meet at right angles.

In his consideration of the problem of the exchange values of goods, he emphasized the interdependence of the quantities involved in his highly simplified market for consumers' goods. Despite his awareness of the unity of his restricted system, he only adumbrated the Lausanne school, but he took two important steps forward. Having made his simplifying assumptions, he set up a hypothetical model, which he used to get at the heart of price determination. Consequently, he was able to arrive at the view that value is not an absolute quantity "contained" in a good but a quantity which must vary as other quantities with which it is related vary. The fact that Isnard's contemporaries did not accept this novel and important conclusion does not diminish its significance as an early contribution to economics.

At the beginning of his analysis Isnard points out that one can compare homogeneous things with reference to either their quantities or their magnitudes, but among heterogenous things it is necessary to find some homogeneous link. The link that he finds is the ratios of exchange that are established among goods in the market. These exchange ratios are ordinarily expressed in terms of money; but, if money does not exist, they can still be stated. For example, given a certain quantity of one good equal in exchange to a certain quantity of another good, a representing the number of units of the first good and b the number of units of the second, one may say that the value of the first good in terms of the second is equal to b/a. And if goods taken two by two have ratios between them, they also have such ratios taken all together. For example, if we have three goods of which the values are M, M', and M'', such that $M : M' :: 1 : 2$ and $M' : M'' :: 3 : 5$, we can write $M : M' : M'' :: 3 : 6 : 10$. These ratios give the value of the goods:

> The word "value" then expresses the ratio between two things when they are compared in exchange. In speaking of economic goods, one rarely uses the word "value" in an absolute sense. The word which properly expresses the absolute meaning which one would like to give to it is "utility."[23]

Now note the questions which Isnard proposes:

> Comment les choses acquierent-elles une valeur dans les échanges? Comment cette valeur dépend-elle de la quantité des choses et du besoin qu'on en a? Comment les quantités dépendent-elles du besoin et des valeurs? Comment les besoins sont-ils subordonnés eux-mêmes aux quantités et aux valeurs? C'est ce qu'il faut découvrir successivement.[24]

At this date (1781) he was asking questions which were not seriously asked again for over half a century.

He considers next how exchange ratios are determined. First, suppose a case in which there are owners of two goods, with each owner wishing to dispose of a part of the good that he owns. If one owner wishes to get rid of a units of good M and the other owner wishes to dispose of b units of M' and since there is a possibility of exchanging only these two goods, then a units of M are given in exchange for b units of M'; hence, we have aM equals bM', and consequently $M : M' :: 1/a : 1/b$. The value of each unit of merchandise is then in inverse ratio to the quantity offered in exchange:

> If, instead of two goods, we suppose three to be exchanged, or a greater number, it will be the same for the general value of the goods. Each unit of a good will be equal to the sum of the offers made by the owners of the other goods divided by the number of units of the good being considered, or, what is the same thing, the values of goods will be directly proportional

[23] A.-N. Isnard, *Traité des richesses* (London and Lausanne, 1781), p. 17.

[24] *Ibid.*

to the sum of the offers and inversely proportional to the quantity of the particular good whose value is being considered. But since the offers are composed of several heterogeneous goods, it is not possible to deduce from the equality, or from the equation of which we have just spoken, the relation between two particular goods; *in order to find the ratio between goods taken two by two, it is necessary to form as many equations as there are goods.*[25]

We may pass now to a somewhat more complicated case:

Let there be three goods to be exchanged one against the other; let there be a units of M, b units of M', and c units of M''. Let the amount a of good M be divided into two parts, am and an, each of these quantities being the amount of M offered by the owner of M in order to get some M' and M''; let the quantity b of good M' be divided into two parts bp and bq, each of these quantities being the amount of M' offered by the owner of M' to get some M and M''; let the quantity c be divided into two parts cr and cs, each of these quantities being the amount of M'' offered by the owner of M'' in order to get some M and M'. These suppositions give three equations:

$$aM = pbM' + rcM'',$$
$$bM' = maM + scM'',$$
$$cM'' = qbM' + naM.$$

One can deduce from these three equations the ratios of the goods taken two by two, and we then have

$$M : M' : M'' :: \frac{r+p-pr}{a} : \frac{s+m-sm}{b} : \frac{n+q-nq}{c}.$$[26]

[25] *Ibid.*, pp. 18–19. The italics are mine.

[26] The three equations are linearly dependent; any one is the sum of the other two. Assuming consistency, the ratio $M : M' : M''$ can be obtained from any two equations—e.g., by solving

$$b\frac{M'}{M} - sc\frac{M''}{M} = ma$$

$$-qb\frac{M'}{M} + c\frac{M''}{M} = na.$$

The consistency of the equations is verified by checking the result.

One can deduce also the value of each good relative to each other and the quantities which each owner will attract in exchange for his own offers. . . .

It is easy to account for variations in values by variations in offers; one can see how, proceeding from the supposition that m equals n, p equals q, and r equals s, to the supposition that the quantities n, q, and s equal zero, how, I say, between the the extreme suppositions, progressive changes in the offers have an influence on all the values; to do this it is only necessary to make different suppositions in numbers in place of the algebraic quantities we have just used, supposing always, as we have done up to now, that the quantities of merchandise remain constant.[27]

These quotations should be sufficient to reveal Isnard's clear idea of the interdependence of economic quantities. Perhaps a word ought to be added regarding the introduction of money into his system:

If there are several goods, M, M', M'', M''', M'''', etc., whose values are known and which are in ratio of a to b to c to d to e, etc., we can compare all the goods to one of them; thus we have

$$M : M' :: a : b,$$
$$M : M'' :: a : c,$$
$$M : M''' :: a : d,$$
$$M : M'''' :: a : e, \text{ etc.}$$

The values of M', M'', M''', M'''' will then be

$$\frac{bM}{a}, \frac{cM}{a}, \frac{dM}{a}, \frac{eM}{a}, \text{ etc.}$$[28]

Thus one commodity in an economy can serve as a common measure of all others, and we can always relate to this one commodity the values of all.

VII

Limitations of space preclude detailed examination of other writers, though the works of some of them would conceivably

[27] *Ibid.*, pp. 20–21.

[28] *Ibid.*

be as interesting to certain readers as those cited.[29] The treatment of economic aggregates by the German, Joseph Lang, suggests the usefulness of mathematics for this type of analysis.[30] He insisted, however, upon dividing the population of an economy into three classes—the agricultural, the manufacturing, and the official (service)—an illogical classification which led to an artificial separation of what we should today consider important totals. When he came finally to make his summations, however, he arrived at expressions which evidenced a preoccupation with the same quantities that were to be considered significant well over a century later. It may be added that he was the first of the writers here studied to evolve a formula which contains all the elements, including the concept of velocity of circulation, of the "equation of exchange."

Those especially interested in public finance may find instructive Condorcet's mathematical recapitulation of Turgot's reasoning regarding the proposition that all indirect taxes should be suppressed and a direct tax on the *produit net* levied instead.[31] Or they may find significant an essay by an anonymous English writer, who calls himself "E. R."[32] This author is concerned with the general problem of the shifting and incidence of taxes, and he treats mathematically and in great detail the effects on an economy of the commutation of tithes. As did Beccaria, he took a problem of considerable complexity, defined his terms, reasoned from "hypotheses framed . . . like what we have reason to believe is the actual state of things," and found an answer. E. R. was perhaps too naïve in his belief that, once general formulas have been evolved, statistical information as needed would be available for application. Nevertheless, his method required him to simplify, to abstract temporarily from reality. A comparison of his analysis of the effects of commutation of tithes with Ricardo's wanderings on the subject seems to justify the mathematical method. Finally, there is an extensive treatment of the problem of financing a war, by an American, Samuel Gale, which indicates a thorough knowledge of the mathematics of investment.[33] This writer was called to the attention of economists some forty-five years ago by E. R. A. Seligman in his article on some neglected British economists. Since his mathematical treatment has no especial modern significance, he is not treated fully here, but his book deserves attention from those who are interested in the history of economic doctrine.

A special place in the history of mathematico-economic literature must be assigned to Nicolas-François Canard, who is certainly one of the best of this early group.[34] Since his work has been adequately treated elsewhere,[35] there is no need here to trace the development of

[29] It will be noted, for example, that the writings of J. H. von Thünen are not discussed. That portion of von Thünen's work which is of greatest interest to the modern reader appeared some twelve years after the close of the period under present consideration. However, in his first volume, published in 1826, he insisted upon an algebraic analysis of the forces which determine price (see J. H. von Thünen, *Der isolirte Staat in Beziehung auf Landwirtschaft und Nationalökonomie* [Hamburg: Friedrich Perthes, 1826]).

[30] Joseph Lang, *Grundlinien der politischen Arithmetik* (Kursk: Carl Langner, 1811).

[31] M. C. Condorcet, *Vie de M. Turgot* (London, 1786), pp. 162 ff.

[32] E. R., *An Essay on Some General Principles of Political Economy, on Taxes upon Raw Produce, and on Commutation of Tithes* (London, 1822).

[33] *An Essay on the Nature and Principles of Public Credit* (London, 1784), esp. pp. 23–35.

[34] *Principes d'économie politique* (Paris: F. Buisson, 1801).

[35] Edgar Allix, "Un Précurseur de l'école mathématique: Nicolas-François Canard," *Revue d'his-*

his "equation of determination," which expresses the equality of the moments of two opposing forces which are always tending toward equilibrium. Almost without exception, economists have criticized him adversely. As Professor Allix has commented, however, Canard has ". . . the unrewarded role and the merit of a precursor."[36] To Canard belongs the credit for first attempting a comprehensive treatment of the fundamental problem of price determination as a study in the equilibria of forces. To Canard likewise must be attributed the first unequivocal attempt at supply-and-demand analysis. Whatever his errors may have been, there was a sufficient contribution in his book to have enabled a reader with great technical training and superior intellectual gifts to achieve a clearer and more accurate exposition of the matters which he considered. Although Cournot in the introduction to the *Recherches* finds fault with him, he states that Canard was the only one of the early mathematical writers whom he had read. In a later work Cournot admits that Canard gave him a "point of departure."[37]

Finally, a word should be said regarding William Whewell. The opinion is prevalent that Whewell was simply a "translator," that his work consisted only of putting Ricardian doctrines into mathematical language. Whewell was a vigorous critic of Ricardo, however, and his refutation of Ricardian wage theory is proof that he was an economic thinker who might have made a great contribution.[38]

A recent writer has pointed out that Whewell was the first author to formmulate a means of measuring elasticity of demand,[39] though, strictly speaking, his measurement was that of "flexibility of price." Had Whewell become seriously interested in economics, he might have written the first great work in pure economic theory. A highly competent mathematician and a professor of moral science at Cambridge, he was equipped by training and temperament to become one of the heroic figures in economics.[40] That he did not become such a figure can be partly attributed to his lack of an abiding interest in political economy. With him it appears to have been a subject which provided a satisfying mental exercise. He simply was not disposed to direct his investigations "architectonically."

VIII

What place in the history of economics may we assign to these men? They are almost unknown. Their names do not appear in the general textbooks in the history of economic thought. Years of attention to their lives and works may gradually yield facts which will link certain of them to the better-known writers and, for that matter, to one another. They stand now, however, as more or less isolated figures who cannot be said to have contributed to a *current* of

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toire économique et sociale, Huitième année, No. 1 (1920), pp. 38–67.

[36] *Ibid.*, p. 40.

[37] A.-A. Cournot, *Revue sommaire des doctrines économiques* (Paris, 1877), p. i.

[38] William Whewell, "Mathematical Exposition of Some of the Leading Doctrines in Mr. Ricardo's 'Principles of Political Economy and Taxation,'" *Cambridge Philosophical Transactions*, IV (1831), 155–98.

[39] D. R. MacGregor, "Marshall and His Book," *Economica*, IX (new ser.; November, 1942), 316.

[40] There are some interesting parallels in the lives of Whewell and Alfred Marshall. A mathematician before becoming interested in the moral sciences, he was, like Marshall, second wrangler (1816) and began his pursuits in economics, as did Marshall, by putting Ricardo into mathematical (and more general) form.

thought because there is no discernible *flow*.

Yet the sporadic nature of their contributions does not detract from their importance as precursors of a pure science. As it was once necessary to recognize the contribution of Cournot and Gossen, it now appears proper to give to the earliest mathematical economists the credit which is due them. They saw more or less clearly what their more distinguished contemporaries could not see at all—the need for a method which would make possible a precise theoretical analysis. But, more than this, they provided the ideas for that analysis which, had they been earlier accepted, might have altered the development of economic thought.

It has been well remarked that ". . . most ideas when they become propelling ideas are not actually new."[41] Regarding economic doctrines, Frank A. Fetter has noted that various ideas reappear ". . . like the pieces of colored glass in a kaleidoscope, the same essentially in detail, but in ever-changing settings."[42] But the kaleidoscopic nature of the early development of economics should not make us overlook the importance of the first contribution, however tentative and uncertain it may have been. Something can be gained by studying ideas at their inception. As Aristotle pointed out, "he who thus considers things in their first growth and origin, whether a state or anything else, will obtain the clearest view of them."[43] The magnitude of the contribution of the thinker who makes the strategic synthesis, the final successful innovation, must not cause us to forget the preliminary accomplishments of lesser men.

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[41] Fritz Redlich, *The Molding of American Banking, Men and Ideas* (New York: Hafner Pub. Co., Inc., 1947), p. 2.

[42] "Lauderdale's Oversaving Theory," *American Economic Review*, XXXV (July, 1945), 283.

[43] *Politics*, trans. Benjamin Jowett (Oxford, 1920), p. 26.

UNION SECURITY AND THE RIGHT TO WORK

JOHN V. SPIELMANS

THE phrase "right to work" is used with several distinct meanings. On the one hand, it refers to a claim to continuous employment which workers have, or are by some people supposed to have, on the state, on society, on industry. On the other hand, the expression is applied to the "right" of workers to be safeguarded against interference by labor organizations with the job they have or could get. But, even in this sense, the phrase retains two meanings. Either it may mean the "right" to work in a struck plant, that is, to be protected against the moral or physical coercion of picket lines or other strike-enforcing devices; or it may mean the "right" of workers not to be debarred from employment under union-security clauses.

It is only with an analysis of the issue under the last meaning of the "right to work" that this note is concerned; for, even in this twofold restricted sense, the question is complex and controversial.

ANALYSIS OF THE ISSUE

Union-security contracts are clauses which may or may not be inserted in a collective agreement between a union and an employer which, in one form or another, require union membership as a condition of employment. Under a "closed shop" the employer may hire only union members; under a "union shop" he may hire anybody, but on condition that nonunion workers join the union within a specified short time; and under the much weaker "maintenance-of-membership" contract only union members are obliged, as a condition of employment, to remain in good standing, while nonmembers are not required to join.

Such contracts—that is, the first two kinds—have two distinct aspects: an inclusive and an exclusive one. They are inclusive inasmuch as they compel workers to join in order to get or hold a job; and they are exclusive inasmuch as they permit the union to exclude workers from employment by excluding them from membership.[1]

The "right to work" is thus affected in a different way under these two aspects. Under the inclusive aspect it is only workers who refuse, presumably on their own decision, to join the contracting union who are debarred from employment; while under the exclusive aspect this can happen to workers to whom the union denies membership. This latter aspect therefore contains elements of power capable of being abused which are not present in the inclusive side. Accordingly, it is the exclusive aspect which raises more serious issues in connection with the right to work. But it is the inclusive aspect which is more basic because it creates the conditions under which alone the exclusion can become operative. Let us consider these two aspects separately.

A. UNION SECURITY BY INCLUSION

The requirement that all workers in the bargaining unit belong to the union gives the union "security" of three kinds of varying social justification: against employers' antiunionism, against workers' nonunionism, and against rival unions.

As regards employers' antiunionism, the fact that a collective bargaining agreement

[1] In the maintenance-of-membership contract the inclusive aspect is hardly present, since it does not compel nonmembers to join. It is further weakened under the frequently used "escape clause," which permits previous members to withdraw before the compulsion to maintain membership becomes operative. The exclusive aspect, on the other hand, is present inasmuch as loss of union standing implies loss of employment.

is being made indicates, of course, that the union has already achieved the basic security of being recognized rather than being disregarded or even persecuted. But such recognition may be precarious unless buttressed by a union-security clause; for without it the employer may—if he feels so inclined—undermine the union's majority status by open or, under the rule of labor relations acts, concealed discriminatory hiring and discharge.

The reason for the unions' demand for this kind of security is thus the traditional hostility of American employers toward trade-unionism—a hostility which, by comparison with other countries, has been unusually prolonged and unusually ruthless, having kept the unions weak and on the defensive and having prevented them altogether in most of the large mass-production industries until the New Deal. This hostility, which as open, active opposition began to be broken only a little over a decade ago, has thus been personally experienced by most of the older labor leaders of today. Naturally, they expect the hostility to reassert itself and to endanger the newly won positions, if given a chance. The emotion-charged concern of labor leaders for the "closed shop"—the term being used generically for various forms of union security—is therefore not merely understandable from their point of view but probably also objectively justifiable.

As to security against workers' nonunionism, the usual reason advanced by the unions is that, since all workers benefit from a union agreement, it is their moral duty to belong to the union. The condition-of-employment clause is regarded as the most orderly and effective way of enforcing this duty.

The security thus achieved lies, first, in the safeguarding of the unions' bargaining status against nonunionism of workers; and, second, in the automatic extension of membership. Where unionization is already strong, "union security" may, indeed, spread it chain-reaction-like to almost the 100 per cent limit. Its real importance, however, is in industries where unionization is still weak; for any little toehold gained for collective bargaining can, with union security, be not only maintained but progressively enlarged through compulsorily growing membership.[2]

The question as to whether and why the compulsion to join remains necessary, despite the fact that the legal guaranties of the right to organize have made it, in general, safe for workers to join the unions, as far as hostile employers are concerned, is a separate one.[3]

Of the objections which have often been raised against compulsion to join unions, many are doubtless losing importance as once widespread malpractices are being brought under control. Indeed, such malpractices as excessive money exactions in conjunction with irresponsible, dishonest, and even criminal union officers, which often made the quasi-legal compulsion to join nothing less than an outrage against social decency, are being increasingly checked through (1) the rapid growth of union-regulating legislation, (2) an alerted public opinion and the watchfulness of a critical press, and (3) a growing sense of responsibility on the part of the international unions and the national federations.

At the same time, the more basic objection that compulsion to join "private organizations" is, per se, to be condemned as repugnant to the ideals of a democracy is doubtless losing in significance as the unions develop more and more from "private organizations" into publicly guaranteed, quasi-public bodies which quasi-officially represent the interests of labor at the bargaining table of industry.[4]

[2] The danger to unionization of the various recent state prohibitions of union-security contracts lies in the fact that in all those states unionization, notably in the agricultural industries, which are prominent in all of them, has been very weak, wherefore these prohibitions are likely to keep them weak.

[3] On this point see my comments in "The Dilemma of the Closed Shop," *Journal of Political Economy*, LI (1943), 118.

[4] Of great importance in this question is, of course, the limitation which the Wagner acts (big

Hence, if one regards representative unionism as socially desirable or even necessary, the unions' demand for "inclusive" security seems justifiable in the measure that employers remain potentially hostile and workers appreciably nonunion-minded.

Much less justified, on the other hand, seems the "security" against rival unions, which compels workers who belong to one union to change their allegiance and join the majority union, frequently their "enemy"; for there can be but small social value in compulsion which promotes not unionism versus nonunionism but merely one union at the expense of another, with frequently no worthier issues involved than petty personal or group rivalries and artificially played-up hatreds. While it can be argued that the elimination of union rivalry in a shop is in the interest of stable and peaceful labor relations, the facts hardly bear this out; for, as likely as not, the suppression of rival unions creates a most unwholesome atmosphere of mutual suspicion and underground intriguing and thus gives rise to a host of thorny problems of its own.

Nor can it be claimed—as in the case of nonunion free-riders—that rival-union members, benefiting from the agreement, have the moral duty to support the contracting union; for as members of bona fide unions they are doing their duty by their class; and as to paying for services rendered, a reasonable "bargaining fee" paid by rival union members (or for them by their union) to the contracting union would doubtless be all that moral or social duty could require from them.[5] Hence all the usual arguments in favor of workers' compulsory membership really seem to fail in the case of rival unions.

And yet it is obvious why tolerance of rival unions is not to be expected from a union able to obtain a union-security contract; for a much bigger issue is involved for them than petty rivalries and narrow loyalties. The unions want all workers, rival unionists as well as nonunionists, as their members not only to have them pay fees and dues but also to have them subject to their powers of discipline. This fact brings us to the second aspect of union security.

B. UNION SECURITY THROUGH EXCLUSION

The security which the unions seek through exclusion of "undesirables" from employment is achieved through a combination of unrestricted union-security contracts with sovereignty over membership rules and policies. When free to deny membership to whomever they wish, the unions can, under an unrestricted union-security contract, debar whomever they wish from employment.

Classes of "undesirables."—The first question which presents itself is: Who are those whom the unions may wish to exclude?[6] In

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and little) imposed by permitting union-security contracts only with noncompany-dominated unions which were the duly designated bargaining agent for the majority of the workers in the bargaining unit.

[5] An arrangement similar to this was suggested in an article by William Leiserson, "For a New Labor Law: A Basic Analysis," *New York Times Magazine*, February 6, 1949. In fact, his suggestion went a step further by contemplating no other obligation to any worker under a union agreement than the payment of a fee to the contracting union, which would thus eliminate any compulsion to join on the part of nonunion workers or rival unionists.

[6] In an article, "Judicial Procedure by Labor Unions" (*Quarterly Journal of Economics*, LIX [1945], 370 ff.), Philip Taft discusses the reasons for which unions, according to their constitutions and by-laws, may and do expel members, judging the justness of these reasons from the standpoint of the legitimate needs of the unions to keep themselves intact and effective. He grants, in particular, that upon joining a union a worker agrees to abide by its rules and regulations and that therefore the union has the right to discipline those who violate such rules, provided that the unions act in good faith pursuant to methods prescribed by their laws, such laws not being in violation of the laws of the land or any inalienable right of the members. The area of my discussion coincides only partly with that of Taft, since my interest is in the exclusion of workers, not just expulsion of members, and in its bearing on the right to work under union-security contracts. Accordingly, my categories coincide only partially with his enumerated reasons for expulsion.

trying to answer this question, it appears that in the case of some categories the primary motive is to eliminate them from employment, with exclusion from the union chiefly a means to that end. In the case of others the primary motive is to keep them out of the unions, with loss of employment more or less incidental. In the case of still others, the whole exclusion device is disciplinary, exclusion from employment rather than from the union rendering the punishment only indefinitely more effective as a deterrent.

Exclusion from employment seems the primary motive in the following classes:

a) Undesirables as competitors on the labor market. This is the typical situation in the closed-shop policies of "closed unions," widespread in certain trades, designed to protect the vested interests of inside groups or mere union cliques.[7]

b) Undesirables because of their race. Here the motive is also largely economic: the fear that the lower living standards of Negroes, Mexicans, and orientals might tend to depress union standards, although a great deal of outright race antagonism and prejudice is, of course, intermingled.[8]

c) Undesirables as occupationally unqualified. This concern of the unions with workers' qualifications to reserve the labor market for those with proper qualifications naturally overlaps management's hiring functions. But in some trades the unions are, or claim that they are, in the best position to exercise this function in fairness to the workers; wherefore this function has often been long in their hands.

Exclusion from the union seems to be the primary motive in the following classes:

a) Undesirables for their political tenets and affiliations. Depending on the prevailing tenor in the union, this may mean, e.g., exclusion of "Socialists" by "Communists" or vice versa; of "Trotzkyists" and "Socialists" by "Stalinists," and the contrary. The practically most important situation in today's American union world, however, is, of course, the exclusion of "Communists" by nonpolitical, "pure" trade-unionists. The reason for their wanting to keep such political elements out of the unions, or at least out of union leadership, is largely genuine concern lest economic trade-union functions be neglected or perverted to political ends. Since the Taft-Hartley Act there has been the added reason not to be cut off from the services of the National Labor Relations Board. Moreover, in view of labor's great efforts to get that act repealed, some of the unions' noisily virtuous Communist housecleaning has doubtless also had one eye on its reassuring effect on the public and on Congress.

b) Undesirables on account of "disloyalty" to the union. Such disloyalty may be no worse than bona fide criticism of individuals and factions in power.[9] More seriously, it means the leaning toward and promoting of rival unions. This situation has taken on special importance under the "majority rule" for collective bargaining in conjunction with the schism in the labor movement: members of a defeated union (say, CIO) who, under a union shop, are forced to join the victorious union (say, AFL) are often suspected, justly or unjustly, of promoting their old union, of betraying secrets to it, or even retaining "dual" membership in it.

A separate group in this "disloyal" class are the active enemies of the unions, employers' spies and stool pigeons, hired troublemakers, and *agents provocateurs*.

[7] This monopolistic exclusion is often supplemented by the device of the "work permit." When temporarily there are not enough union members to satisfy the demand, the unions, rather than admit more members, sell revokable permits to work to nonunion workers.

[8] Sumner Slichter stated recently that the constitutions of over twenty national and international unions excluded Negroes (*The American Economy* [New York: A. Knopf, 1948], p. 49). Since then, under the influence of the antidiscrimination movement, many may have dropped these restrictions.

[9] This point is stressed by Taft, who finds that administration of discipline for "slandering an officer" or for "creating dissension" or "undermining the union" may easily be abused to prevent legitimate criticism of those in power (*op. cit.*, p. 377).

The ability to drive undesirables of these categories from the shop as well as from the union is, of course, highly welcome to a union holding a union-security contract, in order to prevent contagion of members with unwholesome ideas and to protect their own continued majority status.

Principally punitive in character is, finally, the exclusion of workers who violate "union discipline." By this term I mean—leaving out of consideration the more petty violations of rules—*disobedience to the unions' major economic strategy:* that is, violations of strike orders (working contrary to strike orders and striking contrary to no-strike orders and failure to respond to assessments in support of other unions on strike); violations of boycott orders (as to the handling and buying of unfair goods from unfair employers); and violations of the unions' wage-and-hour standards.

To be able through union-security contracts to inflict upon wage-earners the supreme penalty of loss of employment for such disobedience is what gives the unions their true power over the workers: the power to make them do what the unions want them to do and to keep them from doing what the unions do not want them to do; for what greater threat is there to a wage-earner than the loss of his livelihood?

The social justification of excluding "undesirables."—If this analysis is correct, it appears that the social justification of the unions' power over the right to work is neither roundly to be denied nor unreservedly to be upheld. Everything depends upon the classes of "undesirables."

The unions are doubtless justified in ridding a shop of those immoral elements who deliberately and even for pay are causing them harm and discredit. Often also justified is their power to have workers debarred from employment on account of occupational disqualification, where, because of the nature of the trade, they are in the best position to do so.

On the other hand, it seems unacceptable from the standpoint of social justice to accord to self-privileged groups, whether they be exclusive union cliques or "white American labor," a quasi-legal power to debar other groups of workers from employment. (In this respect the "closed shop," under which refusal of membership is prior to the hiring process and thus cuts a worker off from even applying for a job, is doubtless far more oppressive than the "union shop.") Again, it seems peculiarly unjust, first, through a contract clause, to force a man to give up the union of his choice and join another and, next, to drive him out of employment because of his doubtful loyalty to this union-not-of-his-choice. Nor does it seem justifiable to grant to trade-unions the power to drive men out of employment for their political beliefs, free though they should, of course, be to keep them out of positions of union leadership or even out of the unions as long as this does not affect their employment opportunities.

The social justification, finally, of the unions' power over the right to work in order to enforce "union discipline" can, I believe, not readily be assessed on general or ethical grounds. Of course, the procedure ought to be fair and according to prescribed rules for valid "cause" and admit of review by an impartial board. But, as regards the core of the question, the power of the unions to deprive men of their work for failing to do their bidding—this seems to be essentially an issue of conflicting interests only.

To the unions this power to deprive workers of their jobs seems indispensable for the shaping of the amorphous masses of working people into a disciplined army of fighters in labor's unending struggle for a larger share in the social product, for more security, more voice in industry. And eye to eye with them will also be the view of those others who disinterestedly believe that strong unions are essential to the healthy functioning of the capitalist system and that the unions, if they need this power over the right to work in order to be strong enough, should have this power.[10]

[10] "Anyone who recognizes the necessity, desirability, or inevitability of labor unions must also recognize their right to impose sanctions upon their

To employers and other capitalist interests, on the other hand, this power of the unions seems dangerous just because it enhances labor's fighting strength. At least partly on their side are also those who disinterestedly believe that all-too-powerful unions—with their relentless pressing for higher wages regardless of productivity, thus either raising the price level or encroaching on economically necessary profits and restricting management's initiative—are a danger to the healthy functioning of the capitalist system and hence, in the long run, to the workers themselves. This group concludes that the unions should not be allowed to grow too powerful.[11] In agreement with this capitalist viewpoint are many workers who naturally resent being subject to the unions' disciplinary powers. This fact obviously beclouds the issue; for, while this dislike has no more bearing on the social desirability of union power than the dislike of many young men for military service has on the value of "preparedness," it impresses the public and the lawmakers as being relevant; and, in addition, it provides the anti-union side with the noble, humanitarian motive of sponsoring the cause of helpless workingmen and women against ruthless union tyranny.

Thus, while considerable agreement can perhaps be reached on the social justification of the unions' power of exclusion in the case of most classes of "undesirables,"[12] their power to enforce union strategy through the threat of loss of employment is likely to remain, or rather increasingly to become, a mere political issue—that is, an issue of power.

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members who violate rules, or impede their efforts" (Taft, *loc. cit.*).

[11] Ideas to this effect were, for example, expressed in H. C. Simon's "Reflections on Syndicalism": "If unionism were good for labor as a whole that would be the end of the issue for me, since the community whose welfare concerns us is composed overwhelmingly of laborers . . . but . . . there is a conflict of interests between every large organized group of laborers and the community as a whole. . . . The problem is not insuperable as long as unions face substantial nonunion competition. But how does the democratic community limit the demands of strong, secure unions? It is merely silly to complain because groups exercise their power selfishly. The mistakes lies simply with letting them have it" (*Journal of Political Economy*, LII [1944], 4–6). The danger that collective bargaining with powerful unions tends to raise the price level to the detriment of fixed long-term money contracts is frequently emphasized by Slichter (e.g., *op. cit.*, pp. 34 ff.). F. Harbison, on the other hand, in his recent study of collective bargaining in the "power centers" in the automobile industry finds signs of self-correctives developing out of the very bigness of the organizations involved, which tends to widen the outlook of the parties beyond the narrow range of their immediate interests (Harbison and Dubin, *Patterns of Union-Management Relations* [Chicago: Science Research Associates, 1947], p. 197).

[12] Many laws have been enacted and rules adopted to limit the unions' powers to exclude workers from employment: (1) Discrimination on account of race, creed, and national origin is forbidden (*a*) under the several fair employment practices laws, which, among others, make it unlawful for unions to exclude workers from membership and full rights on such grounds (N.Y. 1945; N.J. 1945; Mass. 1946; Conn. 1947); (*b*) under other state laws forbidding unions with discriminatory membership rules to make union-security contracts (Penn. 1939; N.H. 1947; Mass. 1947), or even any collective agreements (Kan. 1941). (2) Unfair exclusion, in general, is restricted under laws authorizing labor boards and commissions to invalidate union-security contracts for unfairly denying membership to employees of the employer (Wis. 1939; Penn. 1939; Colo, 1943; Hawaii 1945) or to exempt unfairly excluded workers from the membership requirements of union-security contracts (N.H. 1947; Mass. 1947). The old Wagner Act NLRB, unauthorized by law, adopted certain rules to protect rival-union workers: under the "Rutland Court doctrine" it forbade dismissal of workers who, toward the expiration of a union-shop contract, changed to another union; and under the "Wallace Corporation doctrine" it forbade discharge of rival-union employees who were refused membership by the contracting union when the employer had previous knowledge of such discriminatory intent. (3) The Taft-Hartley Act indiscriminately denied the unions all powers of exclusion from employment except for nonpayment of regular dues and fees.

RETURNS TO SCALE IN FAMILY FARMING

IS THE CASE OVERSTATED?

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I

ECONOMISTS attempting to study returns to scale in agriculture are confronted with most of the difficulties that perplex the student of this problem in other industries. In addition, there is a myriad of other complexities which cluster around (1) the unavoidably intimate connection of farm production with both the rhythmic and the vagarious biological and climatic processes[1] and (2) the intensely institutional matrix within which the production organization of agriculture is cast. It is upon one aspect of this second circumstance that I should like briefly to focus attention.

The predominant form of organization in American agriculture has been the family farm. The persistence of the term "family farm"—as against such substitutes as "family-size farms" and "typical farm"—suggests its real significance for our problem; for, in spite of the ambiguities of the term and the fact that it has to be redefined for each new purpose, the term persists because it captures the essence of the relationship—viz., the mutuality of the unit of social and economic organization. Family farms are vehicles of family purposes. This is the deeper meaning of the statement that family labor is a fixed factor to most farmers.[2]

Correct insight into the way in which labor is ordinarily incorporated into the economic organization of the family farm reveals that the farmer is usually forced to treat labor as an independent, rather than a dependent, variable in his production decisions. To complicate his problem further, the farmer's labor factor, though independent, is variable over the years: the natural processes of increasing age, the growth and dispersion of his children, the vagaries of accident and illness, keep his labor force in a state of constant, though slowly moving, flux.[3] Couple these facts of life with the relative rigidity in the size of his business, imposed by the line fence and the spatial character of the farm plant, and we can see the core of the farmer's problem in attempting to achieve optimum proportions between the major input factors—labor and land. Essentially, day-to-day and year-to-year production decisions must be caught up in, and subordinated to, the family's program for a career in farming. The past sequence of decisions, arising out of the articulated or nonarticulated objectives in planning their careers, creates the structure of alternatives within which families must choose any immediate course of action.

II

The familial structure of economic organization of agriculture needs to be understood thoroughly if we are to understand the returns to scale in agriculture; for a criticism of the manner in which a given unit of input is used implies the existence of a better accessible alternative. An increase in scale of

[1] Cf. John C. Ellickson and John M. Brewster, "Technological Advance and the Structure of American Agriculture," *Journal of Farm Economics*, XXIX, No. 4, Part I (1947), 827–47, esp. 840–41.

[2] Cf. Theodore W. Schultz, *Agriculture in an Unstable Economy* (New York, 1945), pp. 92–93.

[3] For analysis of empirical relationships underlying this note see a forthcoming University of Wisconsin Research Bulletin, *Labor Foundations of Wisconsin Family Farms*, by Erven J. Long and Kenneth H. Parsons. For a fuller documentation see the writer's unpublished Ph.D. dissertation by the same title (Wisconsin, 1947).

operations is, from the standpoint of the family purposes to be served, simply not an alternative in many cases of small-scale farm enterprises. Frequently that would necessitate passing the farm to some other family or changing the structure of the family-farm institution itself. Even adding a hired man is no easy solution because of the indivisibility of the labor input unit and the cost in social values, as imputed by the family, in surrendering its complete intimacy.

Farm families are, of course, heterogeneous. Some families have several children; some have none. Some families have only sons; others only daughters. Some children are interested in remaining on the farm; some are not. In some families the spread in age between fathers and sons is quite wide; in others more narrow. These heterogeneities have their corresponding reflections in the scale of operations which farmers conduct. Farmers with husky, mature sons operate larger farms than do farmers with no sons to help them. After they pass the prime of life, farmers without family help taper down the scale of operations as they grow older. The first and sharpest cuts are made in the relatively variable factors, such as livestock, special crops, and labor-consuming farming practices. But gradually the realities of life bring about more radical adjustments in the farm business: renters move to smaller farms; owners splinter off pieces of their farms, field-rent some of their land, lease out entire enterprises, sell their farms, and buy smaller ones.

Using the "war unit"[4] as a composite measure of the scale of farm operations (because it had been already computed and was adequate to the task), the writer and his associates found, in a five thousand-farm sample in Wisconsin, that, from an average of twenty-seven units per farm among farm operators in their twenties and thirties with no children over ten years of age living at home, the scale of operations dropped to twenty units for operators in their fifties, seventeen units for operators in their sixties, and twelve units for operators in their seventies. This corresponds directly with the declining average physical capacities of farm men resulting from their increasing age.[5] Farmers who had children at home, but no sons over sixteen years of age, followed the same general pattern; together, these two groups represent over 60 per cent of all the farms in the state. Father-son farms, however, where the father's diminishing physical strength meshed with the increasing age and capacity of the son, experienced great stability in the scale of operations.

III

All the foregoing is, in a general way, commonplace knowledge to farm people and to students of the economics of farm organization. But the precise significance of these variations in physical capacities of different farm families has not been simply formulated, partly because they have never been really considered the basic analytical categories for the investigation of farm organization. As a consequence, I believe, farm economists have injected a consistent bias into their attempts to analyze the returns to scale in agriculture. This bias has tended to run in terms unfavorable to the small-size family farm on at least two counts. It has, by overrating the labor force on small farms, underrated the relative productivity of labor. Also, by failing adequately to analyze the structure of alternatives accessible to the family members on small farms, the opportunity cost of the labor input factor (the only concept of cost really meaningful in this context) has been considerably overestimated.

Conventional analysis falls roughly into

[4] The "war unit" was devised by the U.S. Department of Agriculture for purposes of administering war programs. One war unit is equaled by one dairy cow, three sows to farrow, seventy-five laying hens, twenty acres of corn, etc.

[5] The estimates of nearly five thousand farm operators of their own, and their various family members', capacities for farm work, recorded in a complete farm-manpower survey made in 1943 by the AAA, provide the evidence on this point. Careful checks of these estimates completely verified their reliability.

two categories with respect to the first of the above points: either it completely ignores the relationship of age and such factors to the physical capacities of farm men and women,

TABLE 1

AVERAGE NUMBER OF PERSONS (TOTAL MALE AND FEMALE) OVER TEN YEARS OF AGE PER FAMILY, AND AVERAGE MAN-EQUIVALENT PER FARM, BY CROP ACRES PER FARM, WISCONSIN

Crop Acres per Farm	Average No. of Members in Family over Ten Years Old Living at Home	Average Man-Equivalent* per Farm
0–19	2.4	.90
20–39	2.6	1.14
40–59	2.8	1.29
60–79	3.1	1.34
80–99	3.1	1.57
100 and over	3.2	1.70

*A man-equivalent is defined on the AAA form (Budget Bureau No. 40-STO16-42), from which the basic data were taken, as the "work capacity" of "one able-bodied male adult."

or it attempts to make adjustments for this factor when the situation demands it. From the literature it appears that the latter is definitely the minority case. Even among economists who attempt to adjust for variations in the physical capacities of farm workers, the persistent tendency appears to be an inadvertent underestimating of the impact of age on men's physical abilities.

The usual procedure followed by research workers in collecting primary data on farm manpower is to assign to the operator a man-equivalent rating of 1, regardless of his age or health (except in the most extreme cases), and then to rate the capacities of other family members by comparing them with that of the operator. On the other hand, economists analyzing relative net incomes of large and small farms from secondary data usually do not have access to information on labor input units as related to size of farm. The erroneous conclusions to which these procedures might lead can be imagined by examining the relationships in Table 1, taken from a five thousand-farm sample of Wisconsin farms.

It is apparent from these figures that size of farm is positively related both to the size of family and to the amount of family labor per farm. (The 12 per cent of the farms in the state that hire regular labor were omitted.) The fact that the man-equivalent per farm increases much more sharply than does the number of persons per family indicates that the labor capacity *per person* also increases with the size of farm. This was found to be due to two factors: (1) the proportion of children who are mature sons is much larger on the larger farms than on the smaller farms, and (2) a disproportionately large number of small farms are operated by older men.

On farms where mature sons assist the operator, failure to make allowance for differences in physical capacities of operators due to differences in their ages introduces no bias, since there appears to be no consistent relationship between size of farm and age of operator on these farms. But on farms where there are no mature sons, even though there may be younger children, following conventional analytic procedure introduces a very strong bias. Among the Wisconsin farm families who had no children over ten years of age working at home, for example (which

TABLE 2

RELATION OF AGE OF OPERATORS HAVING NO CHILDREN OVER TEN YEARS OLD TO SIZE OF FARM, WISCONSIN

Crop Acres per Farm	Per Cent of Operators under Forty Years Old	Per Cent of Operators over Sixty Years Old
0–19	27.0	30.7
20–39	39.1	24.3
40–59	48.1	14.6
60–79	56.2	12.3
80–99	61.2	11.7
100 and over	60.0	5.9

represents 36 per cent of all farms in the state and over half the families on the smallest 40 per cent of the farms), we find the smaller farms predominantly operated by older operators, with consequent lower physical capacities for farm work (Table 2). This

same general relationship holds for farms where there are children over ten years old but no sons over sixteen.

Calculation by conventional procedure would overrate the man-equivalent on the smallest group of farms by about one-fourth, if the man-equivalent of only the operator were not adjusted for age. But, as would be likely, if the farm-labor contribution of the wife also were not adjusted for her age, the man-equivalent on these farms would be overrated by one-third. For the second size group of farms, the total manpower would be overrated by 16 per cent if only the operator's physical capacities were not adjusted, and 21 per cent if his wife's contribution, likewise, were not adjusted for her increasing age. These figures indicate the magnitude of the overstatement against the small family farm when the more careful of common techniques of analysis are used. In the majority of uncritical comments on returns to scale, even less care is exercised—no allowance being made even for the greater number of children and of mature sons and, in some cases, even of the frequency with which labor is hired on the larger farms. A further bias is introduced if my tentative conclusion from another study is true: that, even on full-time farms, the amount of time spent by the operator in off-farm work is inversely correlated with the size of farm.

More important than any of these considerations, perhaps, is that which emerges from a study of the reasonable alternatives for employment open to the types of families and individuals who operate small family farms.[6] Input units are not alone in being indivisible; acceptable opportunities for employment usually come only to those who can furnish a full man-equivalent. If the labor inputs on small family farms—with their high proportions of aged or disabled operators and broken families—were appraised according to their opportunity costs, the efficiency of these farms in returns per unit of labor input cost might rank surprisingly high. The entire range of problems surrounding the adequate and realistic appraisal of the value of labor input is without doubt the weakest link in research on returns to scale in agriculture.

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[6] Walter W. Wilcox and L. John Kutish, in *Small Farms in Wisconsin* (Wisconsin Agricultural Station Bull. 473), note that "... retired or semi-retired farm families accounted for 53% of those on farms with less than 40 acres of cropland and 27% of those on 40–59 crop acre farms—or an average of 40% of the small farms" (p. 7).

BOOK REVIEWS

Trends in American Capitalism. By the LABOR RESEARCH ASSOCIATION. New York: International Publishers, 1948. Pp. 128. $1.50.

This book is a welcome sign that Marxian scholarship in this country is beginning to grow up. In the past, there have been very few adequately trained social scientists who have written as Marxists, and what studies have been produced have usually been in the nature of *ad hoc* political pamphlets. This book, by way of contrast, is clearly the work of economists who not only know their Marx but who also know the theoretical literature of bourgeois economics and who, at the same time, have a solid grounding in statistics and economic history.

The book covers a good deal of ground, considering that it contains just over one hundred pages of text. The titles of the eight chapters give a good idea of the subjects which the authors have been interested in exploring: "What Is Capitalism?", "Measurement of Surplus Value," "Profits of U.S. Corporations," "Surplus Value in American Manufacturing," "The Organic Composition of Capital," "The American Standard of Living," "Business and Government Spending," and "Postwar View of American Capitalism." Needless to say, none of these topics is treated exhaustively, but all are treated with a rewarding freshness and originality. Since the book is written primarily for nonexperts, even those economists who are entirely unfamiliar with the basic Marxian literature should have little difficulty in following the authors' reasoning.

To me, the most interesting chapter is the second, entitled "Measurement of Surplus Value." Here the authors have set themselves the task of discovering how the product of capitalist industry is divided between the workers who produce it and the capitalists who claim a share by virtue of their ownership of the means of production. At first sight, this seems like a relatively simple problem; but, as the authors show, it is, in fact, a very complicated problem, the solution of which is beset by many theoretical and practical difficulties. Perhaps the greatest of these difficulties is to determine who are the productive workers. Orthodox economics gets around this by making the general assumption that money payments are proof of an equivalent productive contribution. This is certainly a very convenient assumption, but it leads to conclusions which are often repugnant to common sense and can be very misleading as to the real productive capacity of an economy. For example, nearly everyone would agree that there is a very real and important sense in which the worker who turns out automobiles on the assembly line is productive, while the worker who turns out advertising copy designed to assure that Model A gets a larger share of the market than Model B is not productive. It follows, of course, that if the writer of advertising copy could be shifted to the assembly line (or to some other productive pursuit), the "real" national product would be increased. Orthodox national-income theory—which in the final analysis is based on the subjective theory of value—ignores this problem. Marxists—basing themselves on the labor theory of value—cannot ignore it. It must be admitted that, by refusing to ignore it, Marxists get themselves into a whole maze of tricky questions. But these questions are genuinely important, and orthodox economics would add greatly to its usefulness if it would frankly face them rather than assume them away.

There is no space in a brief review to discuss the way these and related problems are handled in the book before us. I imagine that the authors themselves would be the last to claim that they have arrived at definitive conclusions, and it seems to me that on some points they are wrong. (For example, I do not see why profits in the nonproductive industries should be included in surplus value, while wages are excluded. In Marxian theory, both are derived from surplus value created in the productive industries. The authors, however, seem to be unwilling to admit that some workers can live on surplus value produced by other workers.) But, however incomplete the analysis and whatever its shortcomings, there can be no doubt that it opens up a field which deserves vastly more study than it has yet received. It would be highly desirable if the authors of this book would themselves un-

dertake a systematic investigation of the problem of national income from the Marxian point of view. An incidental, though by no means unimportant, by-product of such a study would be the light it could throw on the economic theory and the statistical practice of the U.S.S.R. and the other countries in the growing socialist sector of the world.

The last two chapters of the book seem to me to be an excellent brief critique of the liberal (Keynesian) remedies for what Marxists call the contradictions of capitalism. The authors are absolutely right, in my judgment, when they say that "the essential fallacy in the argument of the liberal economist centers about the role he assigns to government" (p. 110). On the other hand, other parts of the book are less successful. For example, the treatment of corporate profits is misleading at more than one point. Stock splitups and stock dividends are not necessarily a "profit concealing device"; normally they are a legitimate accompaniment of the process of internal corporate saving. Thus, it can only cause confusion to say that "profits can be concealed by adding them to the capital of the corporation, either through stock dividends, or by amassing large reserves, or both. Future profits then seem to run at a smaller rate because they are figured on a larger base" (p. 42). This is the way profits are accumulated, not concealed; the future rate obviously ought to be calculated on the larger base; and, finally, the future rate is not necessarily smaller than the original rate figured on the original base. The methods of profit-concealment are real and varied enough without adding imaginary ones.

The book closes with two appendixes of quotations from Marx. The longer appendix contains hitherto untranslated passages from his *Theorien über den Mehrwert* on productive and unproductive labor. If the reader's appetite is whetted for more, as it certainly should be, he will be glad to learn that at long last an English edition of the *Theorien* is now in preparation.

PAUL M. SWEEZY

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Economic Analysis. By KENNETH E. BOULDING. Rev. ed. New York: Harper & Bros., 1948. Pp. xxvi+884. $5.00.

The revised edition of this well-known book contains virtually all of the original edition, plus a considerable amount of new material. The principal addition consists of a new part on "Macroeconomics." In this new part the author tries to compress into about 150 pages introductory discussions of banking, international trade, and business cycles, as well as certain aspects of public finance. In view of this brevity, frequent inadequacies of treatment are not surprising. However, the inadequacies are of a quite unusual kind; they are due, not to the author's ignorance of, or willingness to slide over, difficulties, but rather to a maddening propensity to whet the reader's appetite for ideas and then run on, leaving him frustrated.

In each chapter of Part II (on macroeconomics) there are new and fruitful expository devices. None of them, however, is fully developed, and the separate chapters are not closely interwoven. The instructor will find them a veritable treasure-chest of teaching devices (and of new ideas), but the student is likely to feel as though a powerful searchlight had been turned upon the scene and then turned off again before he could take stock of his surroundings. In his next revision, the author should at least double the length of this section in order to spell out his ideas more clearly. Brevity is a virtue, but this is one of the rare occasions on which it is overdone.

In contrast with Part II, Parts III and IV ("Marginal Analysis" and "More Advanced Analysis") are quite detailed and, in many respects, represent the best textbook treatment available. Chapter xxxi ("Three-Variable Analysis of a Firm") is especially good, and so are parts of chapters xxxiii (especially the sections on saving and the supply of labor services) and xxxiv (particularly on interest theory and the theory of gain from trade). The argument of chapter xxv is an excellent introduction to capital theory from the viewpoint of the firm, but chapter xxxvi ("Equilibrium of an Enterprise in Time") seems somewhat confused.

In this last-mentioned chapter Boulding develops an argument to the effect that a firm should not always plan its input-output stream so as to maximize the present value of anticipated net receipts but should sometimes choose its "investment period" so as to maximize its internal rate of return (the rate at which the firm's capital would accumulate if all earnings were reinvested). At best, the exposition is very unclear, and at times the argument seems simply to be wrong. I suspect that much of the theory can be rationalized on the (useful) assumption that the firm is operating with an absolute limit to the

amount of capital that it has available. However, the author nowhere indicates that he is making such an assumption. As it stands, the argument (especially the sections on "Explicit and Implicit Rent" and "The Rate of Interest and the Period of Investment") appears quite unacceptable.

A few minor slips may be noted. The argument concerning industry stability, in the case of a negatively inclined supply curve (p. 246), is incomplete in that it is not indicated that the equilibrium is either stable or unstable depending upon the nature of the adjustment mechanism assumed. The Wagner Act did not outlaw yellow-dog contracts (pp. 397–98). In Figure 99C the production-revenue contours are not independent of the sales contours (the locus of the "sale-line," however, is not affected by this fact).

Despite these minor criticisms, the book is really an excellent text (or collection of texts for essentially different courses). Part I, little changed from the first edition, provides the student with a really penetrating first glance at the operation of a pricing system. In this part, as throughout the book, the experienced reader will be struck by the ingenious way in which he is constantly made to view familiar things in a new light. Not infrequently, the author's insight leads the student into problems that one would rather have left untouched at the undergraduate level, but perhaps this is a price that must be paid if we are to have texts written by creative minds.

M. W. REDER

Stanford University

The Measurement of Production Movements. By C. F. CARTER, W. B. REDDAWAY, and RICHARD STONE. (University of Cambridge, Department of Applied Economics Monograph No. 1.) Cambridge, England: At the University Press, 1948. Pp. vii+135. 12*s.* 6*d.*

This little volume combines the solid content of a first-class treatise on how to construct production indexes with the human interest inherent in a candid sketch of the trials and tribulations—and confessions—of those who have just completed one. It is written by the men who sweated through the construction of the new London and Cambridge Economic Service Index of Industrial Production. They seem to have had as much fun telling their story as would a member of the Explorers' Club with wounds to exhibit.

The monograph does three things. It discusses the principles and problems generally involved in measuring production movements, illustrating the points from the experience with the London and Cambridge index; sets forth the components, weights, and other details of the new index; and gives a useful summary of the historical statistics of industrial production in the United Kingdom.

Everything of practical importance is treated in the discussion of method. The range of economic activities that should or can be covered, the difference between measuring production when completed and as the work is done, interindustry and temporal variation in the relation between net and gross output, problems in the selection of series to represent an industry's output (involving such matters as degree of detail, sampling, and the use of deflated values, input of materials, or input of labor to measure physical quantities), the choice of time unit (particularly troublesome when short-period changes in production are to be measured), the question of weights, the effect of different base years, and the use of preliminary figures in an up-to-date index with consequent need for revision—all these problems are examined and the practical principles evolved in handling them presented for the guidance of others.

The discussion is especially to be commended for three virtues rare in statistical (or other methodological) treatises. It puts emphasis on the important questions, for example, whether the available series really measure the production changes they are supposed to represent rather than on, as the authors put it, "an elaborate discussion of weighting formulae and the like." It reveals clearly the compromises needed to develop practical working principles out of theoretical ones and to adjust concept and data to each other. It shows frankly to what a considerable extent "acts of faith" and "guesses" (the usual euphemisms, "assumptions" and "estimates," the authors have consciously tried to avoid) go into constructing an important class of "objective" economic measurements.

The men who direct the preparation of our own monthly indexes of industrial production will find little in this methodological discussion that they do not already know. But I am sure they will enjoy reading it as much as I did and will want copies of the monograph for the newer

members of their staff. It is a lively, clear distillation of the experience accumulated since the publication (in the *American Economic Review* for 1921) of Walter W. Stewart's article on "An Index Number of Production." It should prove invaluable to teachers and students of statistics, users of price as well as production indexes, and—especially—that expanding group of economists and statisticians whose countries are only now discovering that they need to measure changes in business activity.

SOLOMON FABRICANT
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Our National Debt: Its History and Its Meaning Today. By the COMMITTEE ON PUBLIC DEBT POLICY. New York: Harcourt, Brace & Co., Inc., 1949. Pp. xviii+182. $2.50.

The Committee on Public Debt Policy was organized under a grant from the Falk Foundation, with a membership and advisory group from industry, finance, and insurance and from university faculties. This book is an edited consolidation of seven monographs put out by the committee between 1946 and 1948; but the Foreword states that the chairman, Mr. W. Randolph Burgess, "rather than the full Committee takes responsibility for the final editing."

The first three chapters trace the growth of the national public debt on a per capita basis and as a percentage of national income and the changing relationship between national income and interest on the debt. The discussion stresses the importance of the wartime expansion of the monetary supply as a "basic cause" of rising prices and the limitation that the needs of wartime finance in general, and particularly the desire of the Treasury to hold down interest rates and to stabilize bond prices, has had upon the freedom of action of the Federal Reserve System.

The next three chapters deal with the relation of the debt to interest rates, savings, and life insurance. The discussion of interest rates starts out as a judicious criticism of the Treasury policy of setting an artificially low rate but turns into a defense of high interest rates. It is one thing to take issue with a Treasury policy of stabilizing interest rates at a low level, particularly when the policy contributes to inflation; but it is a different matter to give such a black picture of the economic effects of low interest rates as is given in this report (pp. 89–93). "A fair reward" to savers is a "human right" (p. 96); and the "falling interest rates" of recent years are among the "gross inequities" that "threaten the independence of privately financed organizations and discourage savings needed for economic development" (p. 143).

The terminology of national income is handled carelessly, and some of the comparisons of national income and saving are of dubious statistical significance. The term "national income" is used repeatedly, but it appears from the footnote on page 10 that in at least some of the analysis the Department of Commerce series, "Income Payments to Individuals," and not national income was used. Later (pp. 107–8) the discussion is in terms of "gross national income," a term which apparently describes what the Department of Commerce calls "gross national product." But in most places in the volume there is no indication as to whether "national income" means "national income," whether it means "income payments to individuals," or whether it means "gross national product." The term "saving" is used repeatedly without definition, but the charts and tables in the latter part of the volume are in terms of "gross savings," which are not savings at all but a combination of savings, expenditures for replacements, and allowances for depreciation; and, whatever the usefulness of the concept may be in discussing the market for capital goods, it is of little use in analyzing the saving habits of the public or the funds available for additions to capital.

The concluding chapter advocates (pp. 147–65): (1) more effective control of government expenditures; (2) debt reduction related to the state of business activity; (3) greater distribution of the debt among nonbank investors; (4) restoration of flexible interest rates; and (5) tax reduction.

Many economists who otherwise might agree with these as general policy objectives will probably react against them in the setting of the last chapter. A good deal of this chapter is a general criticism of government spending, high taxes, and low interest rates. "The drain of taxes," "menace of debt," "cancer," "gigantic obligation," "smothering of enterprise," "loss of human freedoms," "gross inequities," "baneful influence," and "dead hand of bureaucracy" are samples of the style of the chapter.

The book has much useful material and gives some needed warnings. In its emphasis on the monetary causes of inflation and on the inflationary potential in budgets and deficits, it helps

to restore a balance in economic analysis of the causes of price increases. Yet the over-all result is disappointing. The economic analysis is careless or defective at too many points, and the last chapter has a tone that seems out of keeping with the spirit of economic research.

FRANK W. FETTER
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The Catholic Church and the Knights of Labor. By HENRY J. BROWNE. Washington, D.C.: Catholic University of America Press, 1949. Pp. xvi+415. Paper, $4.00; cloth, $4.50.

This detailed study of the relations between the Catholic church and the Knights of Labor reveals for the first time the sharp differences among leading Catholic churchmen on the attitude to be taken toward this organization. It was, of course, publicly known that the emergence of this order caused grave concern among the Catholic bishops, for some felt that it should be condemned outright while others opposed such a step. The latter saw no conflict between the principles of the Knights and the dogmas of the Catholic church. Some also feared the effect of a condemnation by the church upon the thousands of Catholic workers who were affiliated with this organization. The Reverend Browne has examined a huge mass of material, including diocesan archives of the United States and Canada, and he shows the changing attitudes of priests and bishops to the order. He is inclined to give us too much detail, but those who will exercise patience and read through his pages will be amply rewarded.

Moreover, much of his material has never been published. Secrecy-oaths and rituals which were, at the beginning, an obligation imposed on members of the Knights of Labor aroused the unanimous opposition of the church. Largely through the influence of T. V. Powderly, the head of the order, secrecy was abolished in 1881. Some of the bishops regarded this step as a maneuver to allay suspicion but one which did not change the essential character of the organization. Radical social doctrines were another cause for fear of the Knights. A split on this question gradually developed within the church, with Archbishop Corrigan of New York and Cardinal Taschereau of Quebec leading the opponents of the Knights and Cardinal Gibbons of Baltimore opposing any prohibition upon the organization. The latter's attitude was based on several grounds. He sympathized with the efforts of the Knights to organize labor, and he also feared that, if the church were too severe, "they [the Knights] would suspect us of riding with the moneyed corporations and employers." He fought zealously against following the action of the Canadian Cardinal Taschereau, who had condemned the Knights of Labor, and in a letter to Rome he maintained that the organization "as it is constituted in the United States cannot and ought not to be condemned unless we wish to expose the Catholic Church to many dangerous losses." It is fascinating to watch the adroitness of Cardinal Gibbons, who, besides being a scholar and theologian, was also no mean politician. While his argument was based on expediency, Cardinal Gibbons was convinced that the Knights were not secret, subversive, or repugnant to the Catholic church. The dispute arrayed powerful American ecclesiastics against each other, but it was Gibbons' views that were ultimately supported.

A reduction of the detail might have improved the study and made it easier for the reader. Nevertheless, Browne's work is important in showing the wide and divergent opinions that prevail among the leaders of the hierarchy upon social questions. No one could contend, after reading this study, that on social policy the Catholic church is in any sense totalitarian. Certainly, on the Knights of Labor opinions among the bishops ranged from warm approval to sharp condemnation. As in our own day, a bishop might on social issues be anything from radical (without being a Socialist) to reactionary. This volume is useful in showing this truth in a concrete instance as well as in revealing the struggle among the holders of different views in this particular case. The author has enlarged our understanding of a significant episode of labor history and clearly reveals the many-sided views that are held on social questions within the Catholic church.

PHILIP TAFT
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England's Climbing Boys: A History of the Long Struggle To Abolish Child Labor in Chimney-sweeping. By GEORGE L. PHILLIPS, with a Preface by ARTHUR H. COLE. Boston: Baker Library, Harvard School of Business Administration, 1949. Pp. 61. $0.75.

The problem of boys and girls of tender ages, forced or enticed (at the ages involved, the issue

is much the same) into the dirty, dangerous, and often deforming work of scraping the insides of narrow flues, is not itself a major one of economic history. Yet it enters into more than one larger question as a revealing feature, even in cases where other illustrations might serve. It is easy to assume that woman and child labor in mines was long neglected because out of the public view, but the sweeps were quite visible and audible. Distinguished voices were raised on their behalf up to the time when the last fatal accident noted by author Phillips was reported in 1875. The practice continued for decades in violation of laws, partly because substitutes were not fully satisfactory in the technical sense, largely because "the Englishman did not wish to change his habit . . ." (p. 2). Some romance seems even to have become attached to these ways—note "The Sweep's Song" in Benjamin Britten's new (1949) opera.

Mr. Phillips has cleaned up a historical question which needed attention. The Hammonds did not make sufficiently clear the place of these abuses in industrialization, including a laissez faire mentality which shrank in horror from interference with a supposed natural law of social relations. In the end, industrialization produced chimneys and appliances for cleaning them which ended the horrors of half-grown human swabs; but the system had been developed by a shift "from vegetal to mineral" which was perhaps the key technical feature of the so-called "Industrial Revolution."

The shift to coal as fuel and rising standards of upper-class comfort narrowed and multiplied flues, which eventually wandered about through the walls of mansions, shops, and tenements. Coal soot formed hard clinkers besides catching fire occasionally. The bends made mechanical cleaning difficult, and the smaller sizes admitted only very small people to put out fires or to scale away obstacles. English legalized custom was quite reluctant even to ban the construction of chimneys unsuited to maintenance by humane methods. The last pockets of resistance to legally prescribed change seem to have been country mansions and institutions rather than great cities.

As its subtitle indicates, this monograph emphasizes the struggle to abolish a peculiarly nasty form of child labor. More could have been done about placing the problem in its larger setting of technical development and social effects. However, this was not the main task which the author set for himself, and he cannot be properly accused of ignoring the relationship between micro-economy and macro-economy. The story of the struggle for reform is told in an interesting and fairly compact way—too compact for a digest here. It makes agreeable and fruitful reading. I take it that this fact, together with a brief description, is what a potential reader has a right to expect from a reviewer.

M. M. KNIGHT

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Business Incorporations in the United States, 1800–1943. By GEORGE HEBERTON EVANS, JR. New York: National Bureau of Economic Research, Inc., 1948. Pp. viii+184. $6.00.

In this valuable volume, Professor Evans summarizes the results of his studies in the history of business incorporation in the United States since the opening of the nineteenth century. Laboriously compiled and presented with care, the statistics here set forth will prove most useful tools in the hands of all future students of the American economy.

The heart of the work is an analysis of the incidence of business incorporation in sixteen representative states. Some series are fairly complete; others cover part of the period only. The sample legitimately emphasizes charters issued under general laws; but there is, as well, an effort to account for incorporation by special act. The material on New England seems skimpy for the years before 1840, but, inasmuch as W. C. Kessler's recently published data for that region (*Journal of Economic History*, May, 1948) corroborates the over-all picture presented by Evans, we may accept the general analysis as valid.

The result is a significant portrayal of the trends in incorporations over almost a century and a half, together with some supplementary data on developments in the volume of authorized stock, on the size of corporations, on their life-span, and on the variations in various fields of corporate enterprise. Evans' most important conclusion is that business incorporations move in long waves which are closely related to fluctuations in the business cycle; "total incorporations move preponderantly in the same direction as business cycles." This correlation is explained by the argument that the corporation was an instrument of entrepreneurial activity, an

explanation that also accounts for "the tendency of incorporations to lead turns in business."

This view is acceptable for almost the whole of the period. But it does not hold for the period of origins. In this respect the organization of Evans' material is not altogether happy. The only dividing point he recognizes is 1875; he treats the whole span to that date as if the corporation were then a stable, constant institution. Yet, midway through those seventy-five years, the corporation passed through a radical change which altered its relationship to the economy and therefore to the business cycle. The factors which operated on it later were not the same as those which influenced it earlier.

In the first decades of the century, entrepreneurial considerations, embodied in the will of the incorporator, were not the determinants of the rate of incorporation. The corporation was still a state agency; political and other noneconomic influences had a decisive effect on these fluctuations. Comparison of the case of railroads after 1830 (Chart 4) with that of turnpikes before that date (Chart 2) is thus enlightening. In the case of the former, where general economic factors were dominant, the curves of all states move closely parallel to each other; in the latter, where local political factors were dominant, the parallelism does not exist. The predominance of political and social factors also accounts for the fact that, until 1830, there was a close correlation between the rate of chartering business and nonbusiness corporations.

This deficiency has historical significance, but it does not detract substantially from the magnitude of Evans' achievement. He has grappled successfully with a most difficult problem and has left all students of American economic development in his debt.

OSCAR HANDLIN

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American Transportation in Prosperity and Depression. By THOR HULTGREN. ("Studies in Business Cycles," No. 3.) New York: National Bureau of Economic Research, Inc., 1948. Pp. xxxiii+397. $5.00.

This compact volume is one of the National Bureau's series, "Studies in Business Cycles," and is the first of a projected group of substantive monographs. It is indeed quite fitting that the initial industry study should deal with railroads, which were the first of our major industries to develop a modern pattern of organization involving heavy capitalization, substantial fixed costs, large-scale operation, and highly variable traffic. This industry thus has a particular interest for students of business cycles. Only one short chapter is devoted to highway, water, pipeline, and urban transportation. The book makes extensive use of the complicated statistical reference cycles worked out by the National Bureau, and some familiarity therewith by the reader is desirable, though not strictly necessary. Furthermore, it is almost exclusively concerned with the analysis of cyclical behavior in American rail transportation. Hence transportation economists, economic historians, and others who may be interested are likely to find the going quite unfamiliar. The valuable nuggets of fact and interpretation will, however, be of great value to the general professional reader.

What led Hultgren to undertake this study was the opportunity offered by the extensive, long-continued, and relatively uniform statistics on steam railways. Dating as far back as 1882 for traffic for the roads as a whole and still farther back for certain key lines, the statistical material steadily improves in quantity and quality. The author has accordingly processed the more significant time series and has then compared them with the reference periods of expansion and contraction of the National Bureau. Thus there is an elaborate statistical parade of such matters as ton-miles, average haul, passenger-miles, traffic units, net ton-miles per car-hour, stocks of locomotives and cars, freight cars ordered, ton-miles and number of employees, traffic units per man-hour worked, and so on. The book thus consists primarily of a large number of calculations, each one of which is accompanied by a brief commentary. These are logically grouped under the following topics: freight and passenger traffic, equipment utilization, supply of equipment, worker performance, fuel economy, prices and wages, and, finally, cost and profit. The 151 summary tables and 133 well-drawn charts will be for many readers the most valuable parts of the book. Hultgren has undoubtedly provided the most complete summary statistical record of railway performance ever made available.

The author is interested in such questions as

the relationship of current investment to traffic volume, the response of traffic to business activity, the influence of traffic volume on employment, the amplitude of fluctuations and their timing, the effect of maturity on cyclical behavior, and the pattern of income and costs. On such points he draws many conclusions based on his statistical studies. For example, he shows that railroads have lower unit costs at traffic peaks than at troughs but that the turning points in traffic and in costs are not simultaneous. Another conclusion is that rates generally move inversely to traffic. The studies of efficiency in the use of labor are particularly interesting. Clearly shown are the effects of heavy overhead costs and considerable inflexibility in operation, with the resulting wide fluctuations in profits. All this analysis yields no especially startling conclusions to those familiar with the industry, but Hultgren's precise measurement and careful analysis of timing will be of great value.

One is left with the feeling that the commentary does not fully exhaust the voluminous statistical material. Indeed, the book is somewhat severe to read for all but the avid consumer of statistics. The author does not always make clear what is gained from a particular operation. More extensive summaries of the behavior-pattern of railroads might have relieved this situation. Despite these criticisms, which relate mainly to organization and presentation, the book is clearly an important work. It is on such inductive studies that much future economic analysis will rest.

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